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ABSTRACT

The study applied the conventional linear transportation program to the student assignment problem and investigated methods of measuring the achieved level of desegregation. Existing measures of desegregation were analyzed, and two new indexes were developed for use in the present model and probably for other system analytical models designed to deal with the same problem. The study calculated middle school student assignments for Richmond, Virginia, and experimented with "clustering" high school districts for Detroit, Michigan. Linear programing proved to be an applicable tool to handle several desegregation-related planning problems. The study proposes the implementation of the model to determine the desirability of school district consolidation, delineate school attendance zones, compute individual student assignments, analyze school extension alternatives, and to perform interregional comparisons of school desegregation efforts. (Pages 1-100 may reproduce poorly.) (Author)

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Author's Abstract

The study applied the conventional linear transportation program to the student assignment problem and investigated methods of measuring the achieved level of desegregation.

Existing measures of desegregation were analyzed and two new indices were developed for use in the present model and probably for other system analytical models designed to deal with the same problem.

The study calculated middle school student assignments for Richmond (Va.) and experimented with "clustering" high school districts for Detroit (Mi.)

Linear programming proved to be an applicable tool to handle several desegregation related planning problems.

The study proposes the implementations of the model to determine the desirability of school district consolidation, delineate school attendance zones, compute individual student assignments, analyze school attendance alternatives and finally to perform interregional comparisons of school desegregation efforts.

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Final Report

Project No. 2-C-014
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Application of Linear Programming Models to Determine Optimum School Attendance Areas and Busing Schedules, Subject to Varying Racial Composition.

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Preface

When the Office of Education in Philadelphia contracted V.C.U. for this study, background literature in linear programming was the only base upon which research could begin. Since then a rich collection of mathematical models has emerged. Lambda Corporation's Lagrangian optimization method, network flow models at the University of Florida and Rand's unpublished linear programming approach should be considered the most significant competitive efforts.

Among the respective, although unmeasured, merits of the various approaches, this study offers contribution at least in three respects. First, the investigation was based primarily upon the data of Richmond City where busing is a crucial issue; secondly this project has demonstrated that systems analitical adaptation to the racial balancing problem is not as expensive as it has been generally believed; and finally a great deal of inquiry concerning related efforts originated new contacts which made people and institutions involved in this line of research acquainted with one another.

A project of this scope could not have been successfully completed at such a low cost without the competence and enthusiasm of the School of Business research staff and without the helpfulness of other people connected with the project.

I am particularly indebted to the Richmond and Detroit School Boards for their cooperation in providing and clarifying data, to the Lambda Corporation and Social Research Group at G. W. University for the indispensable consultations.

Within the School of Business, I would like to extend my gratitude to Dean J. C. Hall, Dean J. D. Lambert, Dr. N. M. Haymes for their unwavering support. Dr. C. A. Wickersan and Dr. W. A. Thedford have made invaluable professional contributions to the study.

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Peter F. Fogeny

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Introduction

A long prevailing problem of school administrations and governments of various levels has been the delineation of school districts or attendance areas and the scheduling of student busing often with the purpose to achieve racial desegregation.

This project intends to draw the attention of those involved to a fairly simple method which has been frequently applied abroad in determing general activity zones and regional boundaries for relatively small areas. In its suggested form, the method could aid in analyzing and delineating school attendance areas, minimize costs and time involved in student busing, and finally, perform comparison tests measuring the burden of desegregation in various parts of the country.

Description of the Linear Programming Transportation Model

The suggested computation is based on the Linear Programming Transportation Model[1] which minimizes traveling distances, time, and costs subject to measures of capacity and demand.

Originally the model served to clarify decisions on alternative transportation schedules for computer-minded carriers. A trucking company, for instance, lined up origins, destinations, and unit costs as shown below in a simplified version.

We assume that from the garages of the trucking company to the points of "pick up" (origins) distance is either negligible or its minimization is of no interest. (It will be seen that the extension of the model to include "from garage to points of pick up trips" is easy.)

points of pick-up / points of delivery	Pine Street	South Street	Total Capacity
Marshall Avenue	$ .50	$ .25	50
Mosby Circle	$ .10	$ .05	60
Total Demand	40	70	110

In the last column the distribution of capacity is shown. The Marshall Avenue factory or warehouse has 50 units and the one at Mosby Circle has 60. The last row indicates the volume of delivery requirements at department stores (which might serve as destinations). The amounts

required at the destination points and those to be delivered at the origins are equal.

The cells "inside" the matrix indicate unit costs of shipping for the company. $.50, for instance, is the unit costs of delivery from Pine Street to Marshall Avenue. Whatever the transported goods, it has to be homogeneous or else amounts of that good would not be additive and unit costs of its shipping would have no meaning. After going through the red tape of linear programming, the results can be summarized in a so-called optimal table:

points of pick-up / points of delivery	Pine Street	South Street	Total Capacity
Marshall Avenue	0 $ .50	50 $ .25	50
Mosby Circle	40 $ .10	20 $ .05	60
Total Demand	40	70	110

The bracketed amounts in the North-East corner stand for the assignments of a given origin to a destination. 20, for instance, means that 20 out of the 60 units to be delivered at Mosby Circle will have to come from South Street in order to maintain the minimum cost schedule.

---

[1] Hitchcock, F. L., "Distribution of A Product from Several Sources to Numerous Localities", Journal of Mathematical Physics, Vol. 20, and Koopman, T. C., "Optimum Utilisation of the Transportation System", Econometrica, Vol. 17, Supplement, 1949.

The minimum total cost is calculated in the following way:

0 x $ .50 + $ .10 . 40 + $ .25 . 50 + $ .05 20 = $17.50.

This last row of numbers called the "objective function"[2], has reached the minimum value as a result of linear programming iteration. The real-life version of the model involves thousands of origins and destinations necessitating the use of computers. Programs for the transportation model are widespread.

The analogy of the above trucking problem with that of student busing is clear: There are schools (destinations) and localities or census blocks (origins).

The same model for attendance areas would appear as in figure 1.

Localities / Schools	1	2	. . .	j	. . .	25	Capacity
1	t_{11}	t_{12}	. . .	t_{1j}	. . .	t_{125}	A_1
2	t_{21}	t_{22}	. . .	t_{2j}	. . .	t_{225}	A_2
.	.	.	.	.	.	.	.
i	t_{j1}	t_{i2}	. . .	t_{ij}	. . .	t_{125}	A_i
.	.	.	.	.	.	.	.
10	t_{10}	t_{102}	. . .	t_{10j}	. . .	t_{1025}	A_{10}
Demand	a_1	a_2	. . .	a_j	. . .	a_{25}	$\bar{a}$ / $\bar{A}$

Figure 1. The model with symbols.

t_{ij} is the distance (expressed in time, distance, or traveling cost) from the i^{th} school the j^{th} locality.

With the application of the linear programming technique it is assumed that the assignment of students from localities to schools will result in a minimum distance (time or cost) to be traveled by the student population of the Study Area (Area).

The applied linear programming procedure will assign every student to a school and will utilize all the available seats within the program.

If every locality has been assigned to a given school, then school district boundaries or attendance areas can easily be drawn from the optimal table, and this will determine a busing schedule based on the physically most desirable attendance patterns.

The consideration of purely physical factors, like the distance from a census block or locality to a school, the capacity of schools in the Area, and demand toward that capacity, furnishes a good starting point to develop attendance areas, devoid of politics and other complicating factors. But also, there are politics, and the "spatially optimal" pattern will be subject to changes resulting in higher commuting time and costs than those which the proximity and capacity figures suggested.

Desegregation objectives of school boards and governments can be built into the model by dividing each school's capacity into black and white subcapacities. The result will show the minimum commuting time or costs subject to desired racial composition, which will be a greater number than the original minimum based on purely physical factors. Here a test is facilitated on the deviation between the physically most desirable attendance areas and those which resulted from a combination of physical and political factors. The sacrifice for the sake of political expediency can be measuredin terms of time and cost. If the method isapplied in two different regions, then a comparison of the extra busing burden is possible.

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[2] The model's objective function is expressed as $\sum_{i=1}^{10} \sum_{j=1}^{25} ti_j \, Xi_j \rightarrow \min$ subject to $\sum_{j=1}^{25} Xi_j \leq a$ and $\sum_{i=1}^{10} Xi_j \geq d$, where Xi_j is the number of students assigned to i^{th} school from j^{th} locality.

For the application of the model, the following initial data should be determined:

FIRST, the capacity of each school in the area with which the computation is concerned. Capacity may be best expressed by the number of students which each school can accommodate. Capacity should be broken down by grades, races or by other factors of classification. (These numbers will form the last column.)

SECOND, the demand for schools in the area with which the computation is concerned. The demand is expressed by the number of students, using the same grade breakdown and other classifications as above to be accommodated by the school capacity of the community. Demand should be grouped into small area units like census blocks, or a group of census blocks if students there use the same bus stops. A small suburb might form such an entity. Standard geographic coordinates maybe used if the number of students can be established for each of them. (The area units used for grouping demand are called the localities, and form the figures of the last row.)

If the number of seats and students are not equal, this may indicate on of the following possibilities:

1. There is a lack of school capacity in the Area, because the number of students exceeds the number of available seats.

2. There is an under-utilization of the available capacity, i.e., the number of students is exceeded by the available seats.

3. The Area under consideration has been delineated in an erroneous way.

Although the program can be computed with ficticious capacities or students, if their real numbers are not equal, the problem of the lack of capacity or the lesser problem of under utilizationwill still exist. As an introduction to the model's use in planning school extensions, or at least in evaluating investment alternatives from the location point of view, the following two notes should be made:

1. The transportation problem does not give heed to areal continuity. The development of "islands", as shown by figure 2, is very likely, and useful at the same time.

Figure 2. The case when "islands" of attraction develop outside the delineated attendance area.

The optimal program may assign localities a_2, a_3, and a_6 to school A_4. The attendance area of A_4 will include the adjacent a_2 and a_3. Locality a_6 had formed an "island" for A_4 within the attendance area of A_5. (This happened because the program minimized student mileage for the entire system.) Quite obviously, a school board will not make students travel from a_6 to A_4 unless desegregation requires it. Rather, such an outcome indicates the necessity of expanding A_5. If there is more than one "island" in the system, the "from island to school" lengths indicate the sequence of desirability in increasing school capacities. Assume that for school A_2 an "island" had developed within the attendance area of A_4 and that that "island" is greater in size than the one shown in the figure (a_6), and that its length from A_2 is also greater than the distance

between A_4 and a_6. In this case, the model indicates that A_2 and a_6. In this case, the model indicates that A_2's expansion is more pressing than that of A_4.

Since school extension is not a goal that may be realized from one day to another, an auxiliary method isoffered for resolving the problem. The number of students in a_6 (see figure 3) can be considered as a part of A_5's attendance area if a similar number of students are released

Figure 3. The exchange of "islands" in order to secure a real continuity with a minimum loss of optimality.

from A_5's and assigned to A_1. This "exchange" of students should be designed in such a fashion that a minimal loss from the optimum solution (the original result of minimum student mileage given by the program) takes place. The "exchange" of areas could be carried out along a straight line connecting A_1 and A_5.

2. The capacity of a school may be reduced by the number of students in its immediate attractive zone.

Capacities may be reduced by the number of students who are located in the immediate vicinity of schools and who, without a trace of doubt, should be served by those schools (or could not be served by any other school in the Area). In figures 2 and 3 the areas delineated around A_4 and A_5 represent the immediate attractive zones. Evidently, traveling distance, time, and cost in getting to school can be neglected from the circled area.

In actuality, the analysis of the concrete socio-economic and geographic conditions, transportation, and other factors is a precondition to the model's meaningful application. Each area and situation differs somewhat, from another, which differences require an analitical flexibility in carrying out and evaluating the computations. These aspects will represent the immediate practical problem of implementation.

With some alterations in the model's technical coefficients, the same basic principles which were applied above for minimizing student mileage could be used to investigate spatial connections in a more complex manner.

Investment costs of school expansion could be evaluated in combination with transportation costs by running the program with technical coefficients as follows:

$$t_{ij} + C_i = t_{i'j}$$

where $t_{i'j}$ is the altered technical coefficients

t_{ij} = the transportation cost of one student from the j^{th} location to the i^{th} school.

C_i = the required investment cost of school expansion for the i^{th} school

The resultant attendance areas compared with those based on transportation costs would give an indication as to the effects of feasible school expansion alternatives, minimizing investment and transportation costs at the same time.

The conversion of variable costs (transportation) into fixed costs (investment) should be executed through the familiar discounting method used in financial mathematics.

With the alteration of the existing school capacities, the effects of alternative school expansion investment plans on the overall students' mileage could be tested.

Programming techniques, related to the alteration of technical coefficients and capacities belong to the so-called "parametric linear programming"[3] realm, which has an extensive literature and computer program library.

There are four different and succesive phases in the implementation of the proposed model:

1. Initial Study - Municipalities are both Origins and Destinations
2. Intermediate Study - School Districts are both Origins and Destinations
3. Final Study - Grade capacities by schools are Destinations and standard geographic coordinates Origins.
4. Individual Assignment Program - classroom capacities are Destinations and individual addresses Origins.

The description of the model that follows will demonstrate only the first three phases.

[3] The major sources in the field are the following: Gass, S.I., and T. L. Saaty: The Computational Algorithm for the Parametric Objective Function, Naval Research Logistics Quarterly, Vol. 2, 1955, and Manne, A.S.: "Notes on Parametric Linear Programming", Rand Report P-468, The Rand Corporation, Santa Monica, California, 1953.

The Calculation of Technical Coefficients

Average distances between schools of various municipalities (sections) in the proposed consolidated area were established including the average commuting distances between the city and the respective counties. A large scale map of the consolidated area was used.

For the average miles between schools within a section, graphs were made indicating all the schools (by number) in that section and the number of miles between them. A random selection of schools was made and distances between each pair was calculated and arranged as shown below. The total number of miles was divided by the number of observations for the average number of miles.

For instance:

Chesterfield County

Total miles 167.75

Observations 15

Average miles 12

	144	182	183	243	244	281
144	x	9	16½	9	5	16
182		x	16	15	12	16
183			x	12½	15½	6½
243				x	4	6
244					x	9
281						x

Considering the large variation in the observed data, to determine the average miles between schools of different sections, an increase of the sample would not have significantly improved the quality of the final figures. Therefore, only a selection of distances were made using shortest through longest observables (9 in each selection).

For instance:

Between sections of Richmond and Chesterfield

Richmond School	to Chesterfield School	Distance
242	244	1 miles
245	244	3 miles
246	243	4 miles
245	144	5½ miles
207	244	5½ miles
242	281	8½ miles
203	281	12½ miles
242	182	13 miles
207	183	19 miles

Again, the total number of miles was divided by the number of observations for the average number of miles. Total miles 73½
Observations 9
Average miles 8

	201	203	204	205	207	208	241	242	245	246
201	x	2	3	1	1	2½	2	5	3½	2½
203		x	2	1½	2½	1½	3	6½	5½	4
204			x	2½	4	1½	2½	6	6½	4½
205				x	2½	1½	2	5½	4½	3
207					x	3	3	5½	3½	2½
208						x	2½	6	6	4
241							x	3½	4½	2
242								x	3½	2½
245									x	2½
246										x

City of Richmond

Total miles 148.25

Observations 45

Average miles 3

Henrico County*

East

Total miles 48¾

Observations 10

Average miles 5

	103	110	205	320	404
103	x	8½	5	6¾	8
110		x	4½	5	4½
205			x	2	3
320				x	1½
404					x

West

Total miles 74⅜

Observations 21

Average miles 4

	109	202	209	309	314	329	331
109	x	3½	1½	4	2	3	3½
202		x	4½	7½	5	1½	1
209			x	3½	1	3½	3½
309				x	2½	6	7
314					x	4½	5
329						x	½
331							x

* Henrico County was done in two graphs (east and west) because of its relation to the city of Richmond geographically.

Between sections of Richmond and Henrico (East & West separately)

Richmond School	to Henrico School (East)	Distance
203	103	2 miles
208	205	2¾ miles
241	110	4½ miles
241	205	4½ miles
204	103	4⅞ miles
206	110	5½ miles
207	110	7½ miles
242	103	8½ miles
245	404	11½ miles

Richmond School	to Henrico School (West)	Distance
203	331	3½ miles
207	329	3½ miles
207	209	4½ miles
201	329	4½ miles
202	203	5 miles
246	209	6 miles
201	309	9 miles
242	109	9 miles
204	314	9¾ miles

Total miles 106

Observations 18

Average miles 6

Between sections of Chesterfield and Henrico (combined east & west)

Henrico School	to Chesterfield School	Distance
110	243	4½ miles
309	144	6½ miles
110	281	7½ miles
103	281	14⅝ miles
404	144	16½ miles
109	281	17 miles
309	183	22⅝ miles
320	182	23½ miles
202	183	24 miles

Total miles 136½

Observations 9

Average miles 15

Initial Study

This phase of the model serves to assess the approximate volume of busing. Based on Richmond School Board data, the following table was assembled:

Origin	Richmond		Henrico		Chesterfield		
Destination	B	W	B	W	B	W	School Seats
Richmond	3	3	6	6	8	8	52,454
Henrico	6	6	5	5	15	15	34,660
Chesterfield	8	8	15	15	12	12	25,318
Students	29,605	17,290	2,805	31,665	2,267	21,796	112,352 / 105,428

Technical coefficents indicate average distances, in miles between and within municipalities. (for instance "3" in the North Eastern corner means that the average commuting distance in Richmond is three miles.) The technical coefficients were calculated by taking the average of several measurements between schools.

The obvious discrepancy between excess school seats over the number of students must be eliminated in order to run the program. One solution to this problem is the installment of a "dummy column". Assignments to the dummy column would, however, result in uncertainties when reading the program. (Ficticious students would have to be distributed by municipalities to the excess capacity of each.)

An alternative solution is the proportional deflation of capacity figures. In order to achieve that, each capacity figure should be multiplied by the "school-system capacity utilization index". The index will be calculated as follows:

$$\frac{\text{Total Capacity - Excess Capacity}}{\text{Total Capacity}} = \frac{112,352 - 6,924}{112,352} = .938372$$

Calculate the actual racial composition accurately.

The ratio of black students in the Study Area is

$$\frac{29,605 + 2,805 + 2,267}{105,428} = \frac{34,677}{105,428} = .329 \text{ Or } 32.89+$$

The revised school capacities should be split according to this figure. Notice in the revised program below that the number of black and white students and that respective subcapacities are equal.

Total black students = 34,677 = Total black capacity = 16,191 + 10,703 + 7,783. The same is true in the relationship of white students and their subcapacity.

The revised program with capacities divided into 70% white and 30% black is shown below:

		Richmond		Henrico		Chesterfield		Dummy	Total
		B	W	B	W	B	W		
Richmond	B	16,191 (3)		(6)		(8)			16,191
	W		17,290 (3)		9,825 (6)		5,915 (8)		33,030
Henrico	B	7,898 (6)		2,805 (5)		(15)			10,703
	W		(6)		21,840 (5)		(15)		21,840
Chesterfield	B	5,516 (8)		(15)		2,267 (12)			7,783
	W		(8)		(15)		15,881 (12)		15,881
Total		29,605	17,290	2,805	21,665	2,267	21,796		105,428

According to the optimal program,

16,191 black students should stay in Richmond

17,290 white students should stay in Richmond

9,825 white students should be bused from Henrico to Richmond

5,915 white students should be bused from Chesterfield to Richmond

2,805 black students should stay in Henrico

21,840 white students should stay in Henrico

7,898 black students should be bused from Richmond to Henrico

2,267 black students should stay in Chesterfield

15,881 white students should stay in Chesterfield

5,516 black students should be bused from Richmond to Chesterfield

The total number of cross-municipality assignment is calculated in the following way:

7,893 + 5,516 + 9,825 + 5,915 = 29,134

From this, the number of students who can be assigned to neighborhood schools should be subtracted. The residual will be the number of students to be bused (Z). Assuming that the cost of busing one student for one school year is around $900, the annual busing bill for the consolidated school system will be $900·Z. (This of course does not cover the expenditures of intra-municipal busing.) If all three municipalities were under desegregation court order, a calculation involving the above number could provide a useful criterion whereby to decide upon the economical character of consolidation.

Expenditures of Extra Busing in Municipality A

\+ " " " " " " B

\+ " " " " " " C

Total Extra Expenditures to achieve racial balance in Study Area

If the above sum is below the inter-municipality busing bill, such as the above $900·Z, consolidation is reasonable. If the above sum is below the inter-municipality busing bill, consolidation should not be recommended.

The Intermediate Study

After the preliminary analysis of the Initial Study, the first phase of actual planning, called "The Intermediate Study", may be executed.

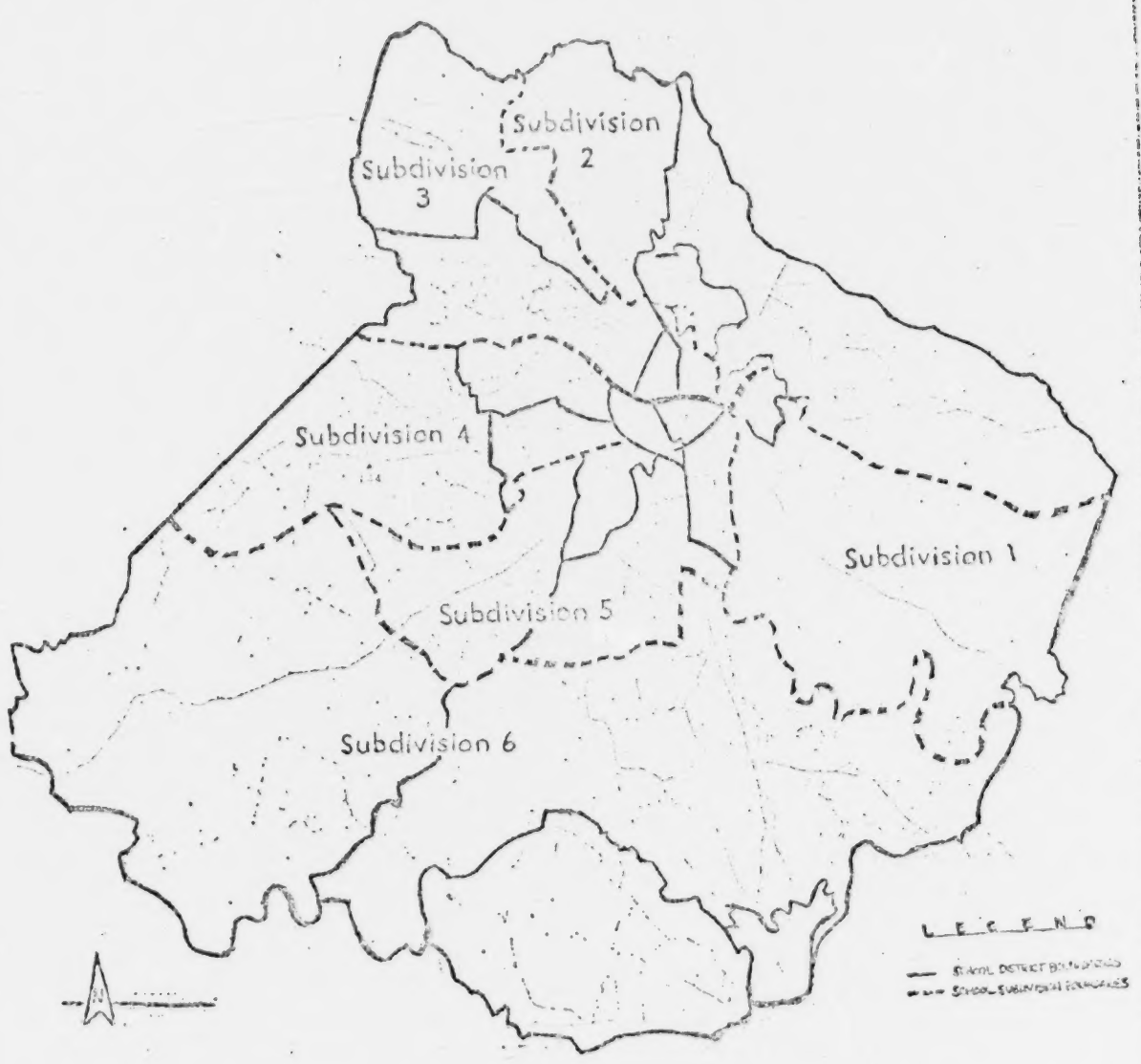
If the Initial Study indicated a need for consolidation in order to achieve a specified racial balance, then as the first phase of implementation the consolidated area should be subdivided into sections. Since the principle of preserving the neighborhood school system coincides with the goal of minimizing transportation, sections should be delineated contiguously and in such a way as to require a minimum cross-sectional assignment. In other words the target racial balance should be assured to the maximum extent in each individual section. The Richmond City School Board has developed a sectional breakdown (sections in the Richmond practice were called subdivisions) exactly with this purpose in mind. The 50 tables which follow have been based on the Intermediate Grades data for the Richmond Consolidated Area. (Study Area).

The intermediate study used sections both for origins and destinations, assuming that in each section there is sufficient capacity to accomodate the section's student population. Destinations, school capacities, had been split according to the existing racial composition of the study area. Technical coefficients were calculated as described before.

The Intermediate Study illustrates the types of problems which may be dealt with at this intermidiary level of analysis.

1. Optimum program, using study area's racial composition as constraint--testing the validity of sectorial subdivision.
2. The effects of changing student population and racial composition upon sections--testing the durability of an optimal sectorial breakdown.

Richmond Study Area



3. The cost effect of an increasing student population.

4. Separation of factors within an overall increase of student transportation bill.

5. Comparing the cost effects of changes in student population with direct increases in transportation costs.

6. Minimizing cross-sectional assignment by using wide racial constraints.

7. Estimation of intra-sectional transportation at varying target composition--preparation for the detailed and individual assignment programs.

* * *

1. Optimum Program, Using Average Racial Composition as Constraint.

Table 1 shows the sectional optimum program for intermidiary grades. The minimum transportation output is 83,034 student miles. This calculation did not alter the reported racial balance.

According to this program neighborhood school systems could be organized by sections. (Assignments in the main diagonal indicate assignment to the same section.) The only exception from this occurs in the case of Section 3 and 4. Students from Section 3 were assigned to Section 4 and vice versa. Since the same cross-assignment persists throughout the entire series of intermediate programs, it is appropriate to assume that further investigation into the section boundaries should bring about improvement in the neighborhood character of study area's sectorial subdivision. It is remarkable, however, that a result as good as the Richmond School Board's sectorial breakdown has been achieved without the application of systems analytical models. The persistent "cross-assignment" between Section 3 and 4 may also be a very difficult planning problem considering that both sections have the same

technical coefficients which does not promise any drastic improvement in the overall transportation output as a result of reshaping the sections.

The program reads as follows:

1151 Black students remain in Section 1

2021 White " " " " "

1153 Black students " " " 2

2312 White " " " " "

1081 Black students of Section 3 will be sent to Section 4

1847 White " " " " " " " " " "

1216 Black " " " 4 " " " " " 3

1745 White " " " " " " " " " "

40 White " " " " will remain in Section 4

1040 Black " " " " " " " " 5

2222 White " " " " " " " " "

313 Black " " " " " " " " 6

1201 White " " " " " " " " "

(The same technique of reading the programs should be applied for consecutive output tables.)

The amount of the so-called cross-sectional assignments can be calculated by summing 1081, 1847, 1216, and 1745 = 5889.

2. The Effects of Changing Student Population and Racial Composition Upon Sections.

Consolidation itself, or any other factors of urban growth, might cause a significant alteration in the student population and in the racial composition of a study area. Such impacts can be and should be studied at the intermediate, sectional level. Suppose that 1a 10% increase in the black population coincides with a 10% decrease of the white population. (This will result in a reduction of the total population, because the absolute number of whites is greater than the absolute number of blacks.) What will be the effect of such a change upon the total volume of student commuting and racial composition? If the change occurred on a proportional basis in each section then both total transportation output and the volume of cross-assignments would decrease proportionally. (Cross-assignment means the number of students commuting from one section to another.)

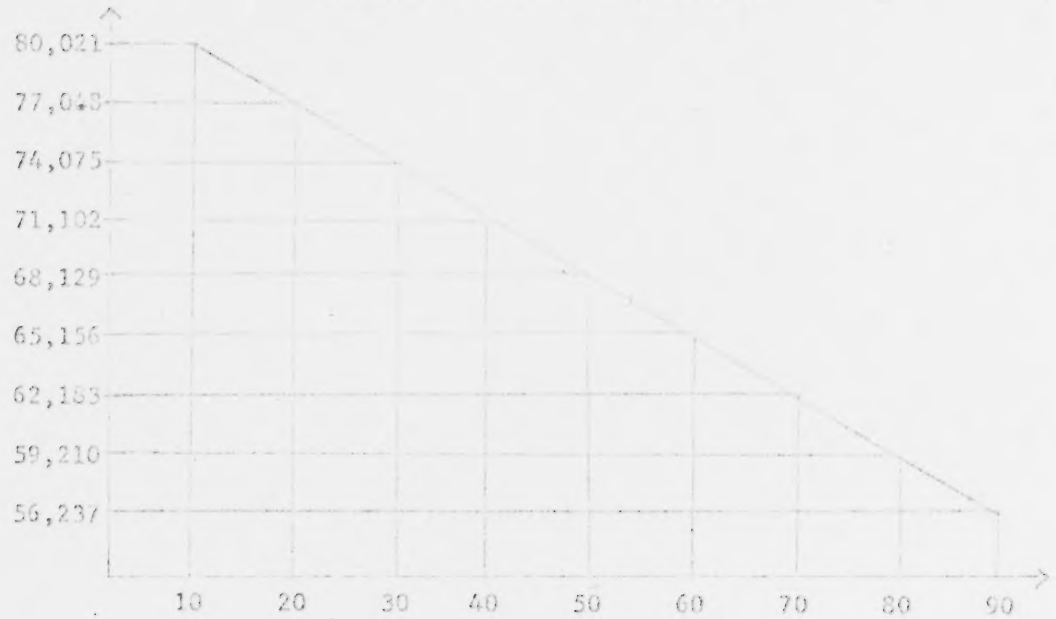
Table 2 through Table 10 shows the reaction of the intermidiary program to changes in racial composition, leaving technical coefficients constant.

Table 2 increased the percentage of blacks by 10% and decreased the percentage of whites by 10% in every section. Table 3 performed the same change with a 20% margin, and so forth. Finally, Table 10 increased blacks by 90% and decreased whites by 90%.

Results are summarized by the following three figures:

Figure 1

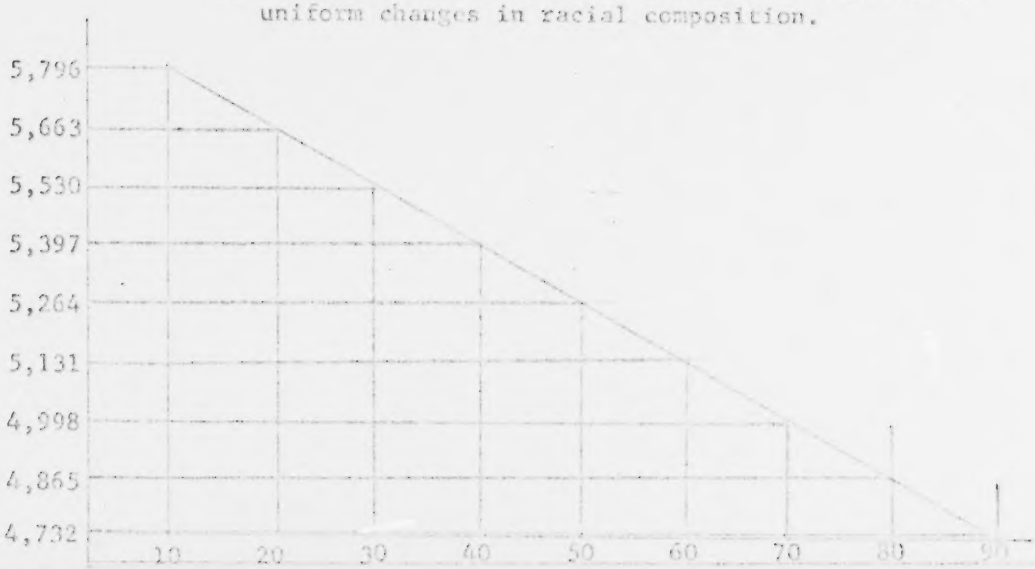
Change in transportation output as a result of uniform changes in racial composition.



Equal reduction in the total number of students, by section, results in a proportional decrease of transportation output regardless of the shift in racial composition. (Evidently, the proportional reduction maintained the originally established target racial composition for each section and for the entire system.)

Figure 2

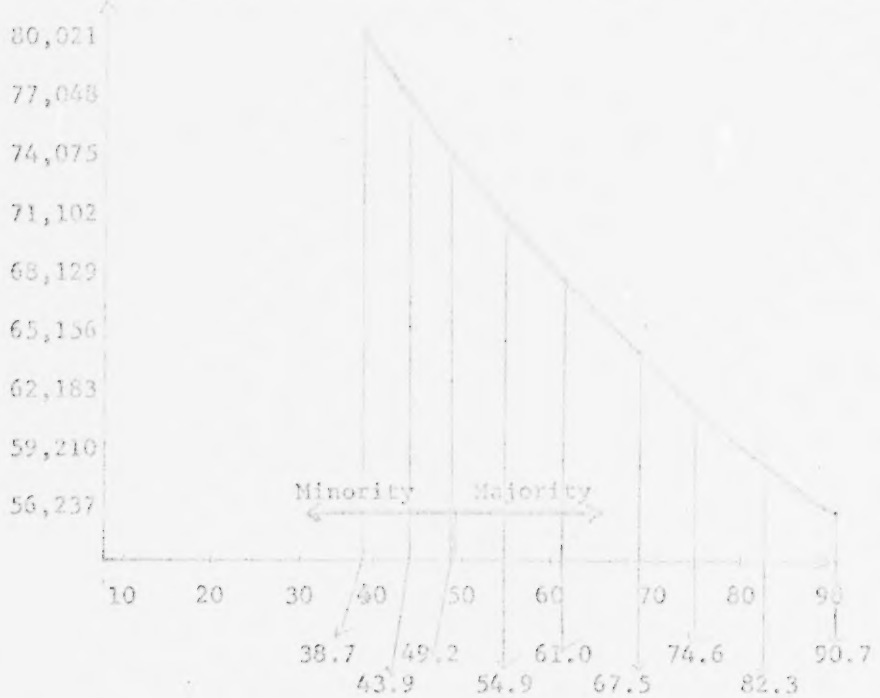
Volume of cross-sectional assignment as a function of uniform changes in racial composition.



Equal reduction in the number of students, by section, results in a proportional decrease of cross-sectional transportation output regardlessof the shift in racial composition. (Evidently, the proportional reduction maintained the originally established racial compotition of students assigned to sections other than where they reside.)

Figure 3.

Increasing % of Black Students in a Diminishing Student Population.



The above curve is downward sloping at a slightly diminishing rate. The lower row of numbers indicate the actual proportion of blacks. Their increasing percentage is produced by the hypothetical changes performed in Tables 2 through 10. The decreasing transportation output is due to a decrease of student population in the study area. Equal reductions in study area's student population in favor of black students and a concommittant proportional reduction in the total transportation output yields a very small response in increasing the percentage of blacks. Consequently, by increasing

the number of blacks and decreasing the number of whites by the same proportion in every section does not decrease the total transportation more than to the extent justified by a decrease of students in the study area. This is so despite an increasing percentage of blacks in the study area. (Except of course, when the percentage of blacks exceeds 50%, thereby becoming majority instead of minority.)

In case of actual implementation, realistic projections should provide the basis for such an investigation. Actual data would, of course, drastically change the linear features of the above three figures whose data have been selected for the sake of convenient demonstration. If, for example, according to reliable projections, blacks will decrease their number by 10% in one section and whites will increase theirs by 20% in another, the change in total and cross-sectional transportation would not be proportional to the overall change in the student population, since the originally established racial composition by sections would have been disturbed. Reliable projections do justify the "resectioning" of a study area. By incorporating future number of students and racial compositions in determing sections, the purpose of minimizing transportation will be better served.

3. The Cost-Effect of an Increasing Student Population.

Tables 11 through 19 demonstrate the fact that, for example, a 20% increase (Table 12) in the capacity and student population of each section does not result in a 20% increase of the total transportation output. Rather, these tables prove that proportional and constant growth of section capacity and student population will increase transportation output and cross-sectional assignments in a predictable way.

In the subject tables the increase in transportation output, as a result of different percentage increases in capacity and student population, remains

a constant; 8,288 student miles. The increase in cross-sectional transportation remains 583 students. When money measure, equivalent to a student-mile, is established, the cost increase of student transportation, in face of a given % increase of student population, may be safely predicted.

4. Separation of Factors within an Overall Increase of Transportation Bill.

Tables 20 through 29 show the reaction of the optimum program to 10% increases in capacity and students and unit increases in each technical coefficient. Table 20, for example, increased capacity and student numbers by 10% and technical coefficients by 1. Table 21 increased capacities and students by 20% and technical coefficients by 2, and so forth.

This type of calculation is very useful in determining the relative weights within an overall increase of the student transportation bill. The total transportation bill and cross-sectional assignments show the following growth patterns:

	Minimum Transportation	%	Cross-sectional Assignment	%
Table 20	110,393		6,477	
21	141,214	128	7,065	109
22	175,497	124	7,630	108
23	213,242	122	8,241	108
24	254,449	119	8,829	107
25	299,118	118	9,417	107
26	347,249	116	10,005	106
27	398,842	115	10,593	106
28	453,897	114	11,181	106
29	512,414	113	11,769	105

The first composite change is reflected by the 28% increase in the minimum transportation values. This number includes both the growth in student population and in individual transportation costs. The residual which remains after separating the approximately 9% growth of students (from 19,073 to 20,804) is the average increase in the costs of transportation:

$$\frac{12\%}{10\%} \approx 117 \qquad \text{(Technical coefficients increases by 17\%)}$$

If a projected time series of student population, transportation cost growth, capacity expansion and any other data of importance are available, the same type of calculation, besides providing the optimum programs, will enable a school board to separate the respective effects of various factors in an overall increase of dollar value transportation output.

5. Comparing the Cost Effects of Changes in Student Population with Direct Increases in Transportation Costs.

Tables 30 through 39 worked with technical coefficients which had been subsequently increased by numbers 1 through 10. At the same time the program varied racial composition in the same way as in Tables 2 through 10: The program increased the number of black students by 10%, decreased the number of white students by 10% in Table 30, and repeated the same with 20% in Table 31, and so forth.

Increasing technical coefficients may represent a worsening of intra-city traffic as expressed by longer continuitytime and higher expenditures. Again, in case of implementation projected changes instead of uniform variation should constitute the base of analysis.

From the augmenting volumes of total transportation a percentage increased series may be formed and that may be compared with the changes in student population as depicted by the subject tables.

Transportation	%	% of Decline in Student Population
96,820		
109,560	113.0	96.7
121,214	110.6	96.6
131,782	108.7	96.5
141,264	107.1	96.4
149,600	105.6	96.2
156,970	104.8	96.1
163,194	103.9	95.9
168,332	103.1	95.8
172,384	102.4	--

From the first two entries of the percentage columns, for example, it is obvious that transportation increased more, (13%) than the student population

declined (100 - 96.7 = 3.3%). Although the applied hypothetical variations may not correspond to reality, it seems likely from these numbers that plausible increases in the costs of transportation would increase the total transportation bill to a greater extent than what likely decreases in the student population could offset.

This sort of calculation may provide the figures to estimate the costs of desegregation in face of simultaneously changing student population and transportation costs.

By separating the two effects, appropriate measures for the reduction of the costs of transportation could be better evaluated.

6. Minimizing Cross-sectional Assignment by Using Wide Racial Constraints.

Table 40 shows the minimum transportation assignment with wide racial constraints. Similar programs can be run if the school board is not tied rigidly to a given racial balance. The program in table 40 considered assignments from 20% Black, 80% White, to 40% Black and 60% White. Interestingly, although not surprisingly, the best composition, from the viewpoint of transportation, is the actual ratio of whites and blacks. Exceptions are the sections from which students had been assigned to other sections. There a racial composition, within the constraints, which will simultaneously minimize cross-sectional assignments will prevail. The cross-sectional figure in Table 40 is 1,159 + 1,785 + 1,081 + 1,827 = 5,756. This number is less by 1,133 students than the optimal program run under the rigid constraints of 30% and 70%.

7. Intra-Sectorial Transportation and Varying Target Compositions.

Since the detailed and maybe individualized intra-sectional assignment is the final product of the entire planning procedure, the intermediate programs has to be able to evaluate the alternative volumes of transportation at

varying target compositions, before the final assignments are being made.

Tables 41 through 50 demonstrate that a variation in section capacity composition from 50% White - 50% Black to 95% White - 5% Black can leave transportation output and cross-sectional assignment unchanged and at the same time change drastically the intra-sectional assignment pattern.

For each alternative target composition the implementing school board will have to analyze the changes which are implied by the program. Naturally, target compositions would not have to vary by the same square pattern as demonstrated by this study. Composition may vary at wider or narrower intervals and would not have to be uniform for each section. When sections are finally determined, planning turns to computing the detailed assignments of the Final Study.

Intermediate Grades Student Assignment Program By Sections.

Minimum Transportation Output: 83,034

Description: Absolute minimum subject to reported capacity.

	1		2		3		4		5		6		TOTAL ASSIGNED	ADJUSTED %
	B	W	B	W	B	W	B	W	B	W	B	W		%
1B	4 1151		7		8		8		6		16		1151	36.3
1W		2021											2021	
2B	7		4 1153		5		6		7		18		1153	35.3
2W				2312									2312	
3B	8		5		4		4 1216		6		16		1216	41.4
3W								1745					1745	
4B	8		6		4 1081		5		6		13		1081	36.4
4W						1847		40					1887	
5B	6		7		8		6		4 1040		13		1040	31.9
5W										2222			2222	
6B	16		18		18		13		13		13 313		313	20.7
6W												1201	1201	
TOTAL STUDENTS	1151	2021	1153	2312	1081	1847	1216	1785	1040	2222	313	1201	17,342	

Cost coefficients printed on form used unless otherwise indicated.

Intermediate Grades Student Assignment Program By Section.

Minimum Transportation Output: 80,021

Description: Increase number of Black Students and capacity by 10%

Decrease number of White Students and capacity by 10%

	1		2		3		4		5		6		TOTAL ASSIGNED	ADJUSTMENT
	B	W	B	W	B	W	B	W	B	W	B	W		%
1B	4 1266		7		8		8		6		16		1266	41.0
1W		1819											1819	
2B	7		4 1268		5		6		7		18		1268	37.9
2W				2081									2081	
3B	8		5		4		4 1337		8		16		1337	45.4
3W								1607					1607	
4B	8		6		4 1189		5		6		13		1189	41.7
4W						1663							1663	
5B	6		7		8		6		4 1144		13		1144	36.4
5W										2000			2000	
6B	16		18		18		13		13		13 344		344	24.1
6W												1081	1081	
TOTAL STUDENTS	1266	1819	1268	2081	1189	1663	1337	1607	1144	2000	344	1081	B Σ:	W

Cost coefficients printed on form used unless otherwise indicated.

Intermediate Grades Student Assignment Program By Section.

Minimum Transportation Output: 77,048

Description: Increase number of Black Students and capacity by 20%

Decrease number of White Students and capacity by 20%

	1		2		3		4		5		6		TOTAL ASSIGNED	ADJUST-MENT
	B	W	B	W	B	W	B	W	B	W	B	W		%
1E	4 1381		7		8		8		6		16		1381	46.1
1W		1617											1617	
2B	7		4 1383		5		6		7		18		1383	42.8
2W				1850									1850	
3B	8		5		4		4 1458		8		18		1458	50.5
3W								1429					1429	
4B	8		6		4 1297		5		6		1		1297	46.7
4W						1479							1479	
5B	6		7		8		6		4 1248		13		1248	41.2
5W										1778			1778	
6B	16		18		18		13		13		13 375		375	28.1
6W												961	961	
TOTAL STUDENTS	1381	1617	1383	1850	1297	1479	1458	1429	1248	1778	375	961		

Cost coefficients printed on form used unless otherwise indicated.

Intermediate Grades Student Assignment Program By Section.

Minimum Transportation Output: 74,075

Description: Increase number of Black Students and capacity by 30%

Decrease number of White Students and capacity by 30%

	1		2		3		4		5		6		TOTAL ASSIGNED	ADJUSTMENT
	B	W	B	W	B	W	B	W	B	W	B	W		%
1B	4 1496		7		8		8		6		14		1496	51.4
1W		1415											1415	
2B	7		4 1498		5		6		7		18		1498	48.1
2W				1619									1619	
3B	8		5		4		4 1579		8		14		1579	55.8
3W								1251					1251	
4B	8		6		4 1405		5		6		13		1405	52.0
4W						1295							1295	
5B	6		7		8		6		4 1352		13		1352	46.5
5W										1556			1556	
6B	16		18		16		13		13		13 406		406	32.6
6W												841	841	
TOTAL STUDENTS	1496	1415	1498	1619	1405	1295	1579	1251	1352	1556	406	841		

Cost coefficients printed on form used unless other wise indicated.

Intermediate Grades Student Assignment Program By Section.

Minimum Transportation Output: 71,102

Description: Increase number of Black Students and capacity by 40%

Decrease number of White Students and capacity by 40%

	1		2		3		4		5		6		TOTAL ASSIGNED	ADJUSTED
	B	W	B	W	B	W	B	W	B	W	B	W		%
1B	4 1611		7		8		8		6		16		1611	57.0
1W		1213											1213	
2B	7		4 1613		5		6		7		16		1613	53.7
2W				1388									1388	
3B	8		5		4		4 1709		8		18		1709	61.3
3W								1073					1073	
4B	8		6		4 1513		5		6		13		1513	57.7
4W						1111							1111	
5B	6		7		8		6		4 1456		13		1456	52.2
5W										1334			1334	
6B	16		18		18		13		13		13 437		437	37.7
6W												721	721	
TOTAL STUDENTS	1611	1213	1613	1388	1513	1111	1709	1073	1456	1334	437	721		

Cost coefficients printed on form used unless other wise indicated.

Intermediate Grades Student Assignment Program By Section

Minimum Transportation Output: 68,129

Description: Increase number of Black Students and capacity by 50%

Decrease number of White Students and capacity by 50%

	1		2		3		4		5		6		TOTAL ASSIGNED	ADJUSTMENT
	B	W	B	W	B	W	B	W	B	W	B	W		%
1B	4 1726		7		8		8		6		16		1726	63.1
1W		1011											1011	
2B	7		4 1728		5		6		7		18		1728	59.9
2W				1157									1157	
3B	8		5		4		4 1821		2		18		1821	67.0
3W								895					895	
4B	8		6		4 1621		5		6		13		1621	63.6
4W						927							927	
5B	6		7		8		6		4 1560		13		1560	58.4
5W										1112			1112	
6B	16		18		18		13		13		13 468		468	43.8
6W												601	601	
TOTAL STUDENTS	1726	1011	1728	1157	1621	927	1821	895	1560	1112	468	601		

Cost coefficients printed on form used unless other wise indicated.

Intermediate Grades Student Assignment Program By Section

Minimum Transportation Output: 65,156

Description: Increase number of Black Students and capacity by 60%

Decrease number of White Students and capacity by 60%

	1		2		3		4		5		6		TOTAL ASSIGNED	ADJUST-MENT
	B	W	B	W	B	W	B	W	B	W	B	W		%
1B	4 1841		7		8		8		6		10		1841	69.3
1W		809											809	
2B	7		4 1843		5		6		7		18		1843	66.6
2W				926									926	
3B	8		5		4		4 1942		8		18		1942	73.0
3W								717					717	
4B	8		6		4 1729		5		6		12		1729	69.9
4W						743							743	
5B	6		7		8		6		4 1664		13		1664	65.2
5W										890			890	
6B	16		18		18		13		13		13 499		499	50.9
6W												481	481	
FINAL STUDENTS	1841	809	1843	926	1729	743	1942	717	1664	890	499	481		

Cost coefficients printed on form used unless other wise indicated.

Intermediate Grades Student Assignment Program By Section

Minimum Transportation Output: 62,183

Description: Increase number of Black Students and capacity by 70%

Decrease number of White Students and capacity by 70%

	1		2		3		4		5		6		TOTAL ASSIGNED	ADJUSTMENT
	B	W	B	W	B	W	B	W	B	W	B	W		%
1B	4 1956		1		8		6		4		16		1956	76.3
1W		607											607	
2B	7		4 1958		5		6		7		18		1958	73.8
2W				695									695	
3B	8		5		4		4 2063		8		18		2063	79.3
3W								539					539	
4B	8		6		4 1837		5		6		13		1837	76.7
4W						559							559	
5B	6		7		8		6		4 1768		13		1768	72.6
5W										668			668	
6B	16		18		18		13		13		13 530		530	59.5
6W												361	361	
STUDENTS	1956	607	1958	695	1837	559	2063	539	1768	668	530	361		

Cost coefficients printed on form used unless other wise indicated.

Intermediate Grades Student Assignment Program By Section

Minimum Transportation Output: 52,210

Description: Increase number of Black Students and capacity by 80%

Decrease number of White Students and capacity by 80%

	1		2		3		4		5		6		TOTAL ASSIGNED	ADJUSTMENT
	B	W	B	W	B	W	B	W	B	W	B	W		%
1B	4 2071		7		8		8		5		16		2071	83.6
1W		405											405	
2B	7		4 2073		5		6		7		18		2073	81.7
2W				464									464	
3B	8		5		4		4 2184		6		18		2184	85.8
3W								361					361	
4B	8		6		4 1945		5		6		13		1945	83.8
4W						375							375	
5B	6		7		8		6		4 1872		13		1872	80.8
5W										446			446	
6B	16		18		18		13		13		13 561		561	70.0
6W												241	241	
	2071	405	2073	464	1945	375	2184	361	1872	446	561	241		

Cost coefficients printed on form used unless other wise indicated.

Intermediate Grades Student Assignment Program By Section

Minimum Transportation Output: 56,257

Description: Increase number of Black Students and capacity by 90%

Decrease number of White Students and capacity by 90%

	1		2		3		4		5		6		TOTAL ASSIGNED	ADJUSTMENT
	B	W	B	W	B	W	B	W	B	W	B	W		%
1B	4 2186		7		8		8		6		16		2186	91.5
1W		203											203	
2B	7		4 2188		5		6		7		16		2188	90.4
2W				233									233	
3B	8		5		4		4 2305		8		18		2305	89.1
3W								183					183	
4B	8		6		4 2053		5		6		13		2053	91.5
4W						191							191	
5B	6		7		8		6		4 1976		13		1976	89.8
5W										224			224	
6B	16		18		18		13		13		13 592		592	33.0
6W												121	121	
TOTAL STUDENTS	2186	203	2188	233	2053	191	2305	183	1976	224	592	121		

Cost coefficients printed on form used unless other wise indicated.

Table 11

Intermediate Grades Student Assignment Program By Section

Minimum Transportation Output: 91,320

Description: Increase capacity and number of Students by 10%

(Ratio of Black and White students remains constant for this series)

	1		2		3		4		5		6		TOTAL ASSIGNED	ADJUSTMENT
	B	W	B	W	B	W	B	W	B	W	B	W		%
1B	4 1266		7		8		6		6		16		1266	36.3
1W		2223											2223	
2B	7		4 1268		5		6		7		18		1268	33.3
2W				2543									2543	
3B	8		5		4		4 1337		8		18		1337	41.1
3W								1920					1920	
4B	8		6		4 1189		5		6		13		1189	36.4
4W						2031		43					2074	
5B	6		7		8		6		4 1144		13		1144	31.9
5W										2444			2444	
6B	16		18		18		13		13		13 344		344	20.7
6W												1321	1321	
TOTAL STUDENTS	1266	2223	1268	2543	1189	2031	1337	1963	1144	2444	344	1321	19,073	

Cost coefficients printed on form used unless otherwise indicated.

Table 12

Intermediate Grades Student Assignment Program by Section

Minimum Transportation Output: 99,606

Description: Increase capacity and number of Students by 20%

	1		2		3		4		5		6		TOTAL ASSIGNED	ADJUSTMENT
	B	W	B	W	B	W	B	W	B	W	B	W		%
1B	4 1381		7		8		8		6		16		1381	26.3
1W		2425											2425	
2B	7		4 1383		5		6		7		16		1383	33.3
2W				2774									2774	
3B	8		5		4		4 1458		8		18		1458	41.0
3W								2095					2095	
4B	8		6		4 1297		5		6		13		1297	36.5
4W						2215		46					2261	
5B	6		7		8		6		4 1248		13		1248	31.9
5W										2666			2666	
6B	16		18		18		13		13		15 375		375	20.6
6W												1441	1441	
TOTAL STUDENTS	1381	2425	1383	2774	1297	2215	1458	2141	1248	2666	375	1441	20,804	

Cost coefficients printed on form used unless otherwise indicated.

Intermediate Grades Student Assignment Program By Section

Minimum Transportation Output: 107,892

Description: Increase capacity and number of Students by 30%

	1		2		3		4		5		6		TOTAL ASSIGNED	ADJUSTMENT
	B	W	B	W	B	W	B	W	B	W	B	W		%
1B	4 1496		7		8		8		6		10		1496	
1W		2627											2627	
2B	7		4 1498		5		6		7		18		1498	
2W				3005									3005	
3B	8		5		4		4 1579		8		15		1579	
3W								2270					2270	
4B	8		6		4 1405		5		6		13		1405	
4W						2399		49					2448	
5B	6		7		8		6		4 1352		13		1352	
5W										2888			2888	
6B	16		18		18		13		13		13 406		406	
6W												1561	1561	
TOTAL STUDENTS	1496	2627	1498	3005	1405	2399	1579	2319	1352	2888	406	1561	22,535	

Cost coefficients printed on form used unless otherwise indicated.

Intermediate Grades Student Assignment Program By Sections.

Minimum Transportation Output: 116,178

Description: Increase capacity and number of Students to 40%

	1		2		3		4		5		6		TOTAL ASSIGNED	ADJUSTMENT
	B	W	B	W	B	W	B	W	B	W	B	W		%
1B	4 1611		7		8		8		6		16		1611	
1W		2829											2829	
2B	7		4 1613		5		6		7		18		1613	
2W				3236									3236	
3B	8		5		4		4 1700		8		18		1700	
3W								2445					2445	
4B	8		6		4 1513		5		6		13		1513	
4W						2583		52					2635	
5B	6		7		8		6		4 1456		13		1456	
5W										3110			3110	
6B	16		18		18		13		13		13 437		437	
6W												1681	1681	
TOTAL STUDENTS	1611	2829	1613	3236	1513	2583	1700	2497	1456	3110	437	1681	24,266	

Cost coefficients printed on form used unless otherwise indicated.

Intermediate Grades Student Assignment Program By Sections.

Minimum Transportation Output: 124,464

Description: Increase capacity and number of Students to 50%

	1		2		3		4		5		6		TOTAL ASSIGNED	ABIDES-NOT
	B	W	B	W	B	W	B	W	B	W	B	W		%
1B	4 1726		7		6		5		6		16		1726	
1W		3031											3031	
2B	7		4 1728		5		6		7		18		1728	
2W				3467									3467	
3B	8		5		4		4 1821		6		16		1821	
3W								2620					2620	
4B	8		6		4 1621		5		6		13		1621	
4W						2767		55					2822	
5B	6		7		8		6		4 1560		13		1560	
5W										3332			3332	
6b	16		18		18		13		13		13 468		468	
6W												1801	1801	
TOTALS ACROSS	1726	3031	1728	3467	1621	2767	1821	2675	1560	3332	468	1801	25,997	

Cost coefficients printed on form used unless otherwise indicated.

Intermediate Grades Student Assignment Program By Section.

Minimum Transportation Output: 132,750

Description: Increase capacity and number of Students to 60%

	1		2		3		4		5		6		TOTAL ASSIGNED	ADJUST-MENT
	B	W	B	W	B	W	B	W	B	W	B	W		%
1B	4 1841		7		8		8		6		16		1841	
1W		3233											3233	
2B	7		4 1843		5		6		7		18		1843	
2W				3698									3698	
3B	8		5		4		4 1942		9		16		1942	
3W								2795					2795	
4B	8		6		4 1729		5		6		13		1729	
4W						2951		58					3009	
5B	6		7		8		6		4 1664		13		1664	
5W										3554			3554	
6B	16		18		16		13		13		13 499		499	
6W												1921	1921	
TOTAL STUDENTS	1841	3233	1843	3698	1729	2951	1942	2853	1664	3554	499	1921	27,728	

Cost coefficients printed on form used unless otherwise indicated.

Intermediate Grades Student Assignment Program By Section.

Minimum Transportation Output: 141,036

Description: Increase capacity and number of Students to 70%

	1		2		3		4		5		6		TOTAL ASSIGNED	ADJUST-MENT
	B	W	B	W	B	W	B	W	B	W	B	W		%
1B	4 1956		7		8		8		6		16		1956	
1W		3435											3435	
2B	7		4 1958		5		6		7		18		1958	
2W				3929									3929	
3B	8		5		4		4 2063		8		18		2063	
3W								2970					2970	
4B	8		6		4 1837		5		6		13		1837	
4W						3135		61					3196	
5B	6		7		8		6		4 1768		13		1768	
5W										3776			3776	
6B	16		18		18		13		13		13 530		530	
6W												2041	2041	
TOTAL STUDENTS	1956	3435	1958	3929	1837	3135	2063	3031	1768	3776	530	2041	29,459	

Cost coefficients printed on form used unless otherwise indicated.

Table 18

Intermediate Grades Student Assignment Program By Section.

Minimum Transportation Output: 149,322

Description: Increase capacity and number of Students to 80%

	1		2		3		4		5		6		TOTAL ASSIGNED	ADJUSTMENT
	B	W	B	W	B	W	B	W	B	W	B	W		%
1B	4 2071		7		8		8		6		16		2071	
1W		3637											3637	
2B	7		4 2073		5		6		7		18		2073	
2W				4160									4160	
3B	8		5		4		4 2184		8		18		2184	
3W								3145					3145	
4B	8		6		4 1945		5		6		13		1945	
4W						3319		64					3383	
5B	6		7		8		6		4 1872		13		1872	
5W										3998			3998	
6B	16		18		18		13		13		13 561		561	
6W												2161	2161	
TOTAL STUDENTS	2071	3637	2073	4160	1945	3319	2184	3209	1872	3998	561	2161	31,190	

Cost coefficients printed on form used unless otherwise indicated.

Intermediate Grades Student Assignment Program By Section.

Minimum Transportation Output: 157,603

Description: Increase capacity and number of students to 90%

	1		2		3		4		5		6		TOTAL ASSIGNED	ADJUSTMENT
	B	W	B	W	B	W	B	W	B	W	B	W		%
1B	4 2186		7		8		8		6		16		2186	36.3
1W		3839											3839	
2B	7		4 2188		5		6		7		18		2188	33.2
2W				4391									4391	
3B	8		5		4		4 2305		8		18		2305	41.0
3W								3320					3320	
4B	8		6		4 2053		5		6		13		2053	36.5
4W						3503		67					3570	
5B	6		7		8		6		4 1976		13		1976	31.9
5W										4220			4220	
6B	16		18		18		13		13		13 592		592	20.6
6W												2281	2281	
TOTAL STUDENTS	2186	3839	2188	4391	2053	3503	2305	3387	1976	4220	592	2281	32,921	

Cost coefficients printed on form used unless otherwise indicated.

Intermediate Grades Student Assignment Program by Section.

Minimum Transportation Output: 110,393

Description: From absolute minimums, increase number of students and capacity 10%; increase coefficient by 1.00.

	1		2		3		4		5		6		TOTAL ASSIGNED	ADJUSTMENT
	B	W	B	W	B	W	B	W	B	W	B	W		%
1B	4 1266		7		8		8		6		16		1266	36.3
1W		2223											2223	
2B	7		4 1268		5		6		7		18		1268	33.3
2W				2543									2543	
3B	8		5		4		4 1337		8		18		1337	41.0
3W								1920					1920	
4B	8		6		4 1189		5		6		13		1189	36.5
4W						2031		43					2074	
5B	6		7		8		6		4 1144		13		1144	31.9
5W										2444			2444	
6B	16		18		16		13		13		13 344		344	20.6
6W												1321	1321	
TOTAL STUDENTS	1266	2223	1268	2543	1189	2031	1337	1963	1144	2444	344	1321	19,073	

Cost coefficients printed on forms used unless otherwise indicated.

Table 21

Intermediate Grades Student Assignment Program By Section.

Minimum Transportation Output: 141,214

Description: From absolute minimum, increase number of students and capacity 20%; increase coefficient by 2.00.

	1		2		3		4		5		6		TOTAL ASSIGNED	ADJUSTMENT
	B	W	B	W	B	W	B	W	B	W	B	W		%
1B	4 1381		7		8		8		6		16		1381	36.
1W		2425											2425	
2B	7		4 1383		5		6		7		18		1383	33.3
2W				2774									2774	
3B	8		5		4		4 1458		8		18		1483	41.
3W								2095					2095	
4B	8		6		4 1297		5		6		13		1297	36.5
4W						2215		46					2261	
5B	6		7		8		6		4 1248		13		1248	31.9
5W										2666			2666	
6B	16		18		18		13		13		13 375		375	20.6
6W												1441	1441	
ACTUAL STUDENTS	1381	2425	1383	2774	1297	2215	1458	2141	1248	2666	375	1441	20,804	

Cost coefficients printed on forms used unless otherwise indicated.

Intermediate Grades Student Assignment Program By Section.

Minimum Transportation Output: 175,497

Description: From absolute minimum, increase number of students and capacity 30%; increase coefficient by 3.00.

	1		2		3		4		5		6		TOTAL ASSIGNED	ADJUSTMENT
	B	W	B	W	B	W	B	W	B	W	B	W		%
1B	4 1496		7		8		8		6		16		1496	36.3
1W		2627											2627	
2B	7		4 1498		5		6		7		18		1498	33.3
2W				3005									3005	
3B	8		5		4		4 1579		8		16		1579	41.0
3W								2270					2270	
4B	8		6		4 1405		5		6		13		1405	36.5
4W						2399		49					2448	
5B	6		7		8		6		4 1352		13		1352	
5W										2888			2888	
6B	16		18		18		13		13		13 406		406	20.6
6W												1561	1561	
TOTAL STUDENTS	1496	2627	1498	3005	1405	2399	1579	2319	1352	2888	406	1561	22,535	

Cost coefficients printed on form used unless otherwise indicated.

Table 23

Intermediate Grades Student Assignment Program By Section.

Minimum Transportation Output: 213,242

Description: Increase students and capacity 40%; increase coefficient 4.00.

	1		2		3		4		5		6		TOTAL ASSIGNED	ADJUSTMENT
	B	W	B	W	B	W	B	W	B	W	B	W		%
1B	4 1611		7		8		8		6		16		1611	36.3
1W		2829											2829	
2B	7		4 1613		5		6		7		16		1613	33.3
2W				3236									3236	
3B	8		5		4		4 1700		8		16		1700	41.0
3W								2445					2445	
4B	8		6		4 1513		5		6		13		1513	36.5
4W						2583		52					2635	
5B	6		7		8		6		4 1456		13		1456	31.9
5W										3110			3110	
6B	16		18		18		13		13		13 437		437	20.6
6W												1681	1681	
TOTAL STUDENTS	1611	2829	1613	3236	1513	2583	1700	2497	1456	3110	437	1681	24,266	

Cost coefficients printed on form used unless otherwise indicated.

Table 24

Intermediate Grades Student Assignment Program By Section.

Minimum Transportation Output: 254,449

Description: Increase students and capacity 50%; increase coefficient 5.00.

	1		2		3		4		5		6		TOTAL ASSIGNED	ADJUSTMENT
	B	W	B	W	B	W	B	W	B	W	B	W		%
1B	4 1726		7		8		8		6		16		1726	36.3
1W		3031											3031	
2B	7		4 1728		5		6		7		18		1728	33.3
2W				3467									3467	
3B	8		5		4		4 1821		8		18		1821	41.0
3W								2620					2620	
4B	8		6		4 1621		5		6		13		1621	36.5
4W						2767		55					2822	
5B	6		7		8		6		4 1560		13		1560	31.9
5W										3332			3332	
6B	16		18		18		13		13		13 468		468	20.6
6W												1801	1801	
TOTAL STUDENTS	1726	3031	1728	3467	1621	2767	1821	2675	1560	3332	468	1801	25,997	

Cost coefficients printed on form used unless otherwise indicated.

Table 25

Intermediate Grades Student Assignment Program By Sections.

Minimum Transportation Output: 299,118

Description: Increase students and capacity 60%; increase coefficient 6.00.

	1		2		3		4		5		6		TOTAL ASSIGNED	ADJUSTMENT
	B	W	B	W	B	W	B	W	B	W	B	W		%
1B	4 1841		7		8		8		6		10		1841	36.3
1W		3233											3233	
2B	7		4 1843		5		6		7		18		1843	33.2
2W				3698									3698	
3B	8		5		4		4 1942		8		18		1942	31.4
3W								2795					2795	
4B	8		6		4 1729		5		6		13		1729	36.3
4W						2951		58					3009	
5B	6		7		8		6		4 1664		13		1664	31.9
5W										3554			3554	
6B	16		18		18		13		13		13 499		499	20.6
6W												1921	1921	
TOTAL STUDENT	1841	3233	1843	3698	1729	2951	1942	2853	1664	3554	499	1921	27,728	

Cost coefficients printed on forms used unless otherwise indicated.

Intermediate Grades Student Assignment Program By Sections.

Minimum Transportation Output: 347,249

Description: Increase students and capacity 70%; increase coefficient 7.00.

	1		2		3		4		5		6		TOTAL ASSIGNED	ADJUSTMENT
	B	W	B	W	B	W	B	W	B	W	B	W		%
1B	4 1956		7		8		8		6		10		1956	36.3
1W		3435											3435	
2B	7		4 1958		5		6		7		18		1958	33.3
2W				3929									3929	
3B	8		5		4		4 2063		8		18		2063	41.8
3W								2970					2970	
4B	8		6		4 1837		5		6		13		1837	36.5
4W						3135		61					3196	
5B	6		7		8		6		4 1768		13		1768	31.9
5W										3776			3776	
6B	10		18		18		13		13		13 530		530	20.6
6W												2041	2041	
TOTAL STUDENTS	1956	3435	1958	3929	1837	3135	2063	3031	1768	3776	530	2041	29,459	

Cost coefficients printed on forms used unless otherwise indicated.

Intermediate Grades Student Assignment Program By Sections.

Minimum Transportation Output: 398,842

Description: Increase students and capacity 80%; increase coefficient 8.00.

	1		2		3		4		5		6		TOTAL ASSIGNED	ADJUSTMENT
	4 B	W	7 B	W	8 B	W	8 B	W	6 B	W	16 B	W		%
1B	4 2071		7		8		8		6		16		2071	36.3
1W		3637											3637	
2B	7		4 2073		5		6		7		18		2073	33.3
2W				4160									4160	
3B	8		5		4		4 2184		8		18		2184	41.0
3W								3145					3145	
4B	8		6		4 1945		5		6		13		1945	36.5
4W						3319		64					3383	
5B	6		7		8		6		4 1872		13		1872	31.9
5W										3998			3998	
6B	16		18		18		13		13		13 561		561	20.6
6W												2161	2161	
TOTAL STUDENTS	2071	3637	2073	4160	1945	3319	2184	3209	1872	3998	561	2161	31,190	

Cost coefficient printed on forms used unless otherwise indicated.

Intermediate Grades Student Assignment Program By Section.

Minimum Transportation Output: 453,897

Description: Increase students and capacity 90%; increase coefficient 9.00.

	1		2		3		4		5		6		TOTAL ASSIGNED	ADJUST-MENTS
	B	W	B	W	B	W	B	W	B	W	B	W		%
1B	4 2186		7		8		8		6		16		2186	36.3
1W		3839											3839	
2B	7		4 2188		5		6		7		18		2188	33.3
2W				4391									4391	
3B	8		5		4		4 2305		6		18		2305	41.6
3W								3320					3320	
4B	8		6		4 2053		5		6		13		2053	36.5
4W						3503		67					3570	
5B	6		7		8		6		4 1976		13		1976	31.9
5W										4220			4220	
6B	16		18		18		13		13		13 592		592	20.6
6W												2281	2281	
TOTAL STUDENTS	2186	3839	2188	4391	2053	3503	2305	3387	1976	4220	592	2281	32,921	

Cost coefficient printed on forms used unless otherwise indicated.

Table 29

Intermediate Grades Student Assignment Program By Sections.

Minimum Transportation Output: [illegible]14

Description: Increase students [illegible] capacity 100%; increase coefficient 10.00.

	1		2		3		4		5		6		TOTAL ASSIGNED	ADJUSTMENT
	B	W	B	W	B	W	B	W	B	W	B	W		%
1B	4 2301		7		8		8		6		16		2301	36.3
1W		4041											4041	
2B	7		4 2303		5		6		7		18		2303	33.3
2W				4622									4622	
3B	8		5		4		4 2426		8		19		2426	41.0
3W								3495					3495	
4B	8		6		4 2161		5		6		13		2161	36.5
4W						3687		70					3757	
5B	6		7		8		6		4 2080		13		2080	31.9
5W										4442			4442	
6B	16		18		18		13		13		13 623		623	20.6
6W												2401	2401	
TOTAL STUDENTS	2301	4041	2303	4622	2161	3687	2426	3565	2080	4442	623	2401	34,652	

Cost coefficient printed on forms used unless otherwise indicated.

Table 30

Intermediate Grades Student Assignment Program By Sections.

Minimum Transportation output: 96,820

Description: From absolute minimum: Increase B 10%

Decrease W 10%

Increase coefficient 1.00.

	1		2		3		4		5		6		TOTAL ASSIGNED	ADJUST-MENT
	B	W	B	W	B	W	B	W	B	W	B	W		%
1B	4 1266		7		8		8		6		16		1266	41.0
1W		1819											1819	
2B	7		4 1268		5		6		7		18		1268	37.9
2W				2081									2081	
3B	8		5		4		4 1337		8		18		1337	45.2
3W						17		1607					1624	
4B	8		6		4 1189		5		6		13		1189	41.9
4W						1646							1646	
5B	6		7		8		6		4 1144		13		1144	36.4
5W										2000			2000	
6B	16		18		18		13		13		13 344		344	24.1
6W												1081	1081	
TOTAL STUDENTS	1266	1819	1268	2081	1189	1663	1337	1607	1144	2000	344	1081	16,799	

Cost coefficients printed on form used unless otherwise indicated.

Intermediate Grades Student Assignment Program By Sections.

Minimum Transportation Output: 109,560

Description: Increase B 20%

Decrease W 20%

Increase coefficient 2.00.

	1		2		3		4		5		6		TOTAL ASSIGNED	ADJUSTMENT
	B	W	B	W	B	W	B	W	B	W	B	W		%
1B	4 1381		7		8		8		6		16		1381	46.1
1W		1617											1617	
2B	7		4 1383		5		6		7		18		1383	42.8
2W				1850									1850	
3B	8		5		4		4 1458		8		18		1458	49.2
3W						74		1429					1503	
4B	8		6		4 1297		5		6		13		1297	48.0
4W						1405							1405	
5B	6		7		8		6		4 1248		13		1248	41.3
5W										1778			1778	
6B	16		18		18		13		13		13 375		375	28.
6W												961	961	
TOTAL STUDENTS	1381	1617	1383	1850	1297	1479	1458	1429	1248	1778	375	961	16,256	

Cost coefficients printed on form used unless otherwise indicated.

Intermediate Grades Student Assignment Program By Section.

Minimum Transportation Output: 121,214

Description: Increase B 30%
Decrease W 30%
Increase coefficient 3.00.

	1		2		3		4		5		6			
	B	W	B	W	B	W	B	W	B	W	B	W		%
1B	4 1496		7		8		8		6		16		1496	51.4
1W		1415											1415	
2B	7		4 1498		5		6		7		18		1498	48.1
2W				1619									1619	
3B	9		5		4		4 1579		8		16		1579	53.3
3W						131		1251					1382	
4B	8		6		4 1405		5		6		13		1405	54.7
4W						1164							1164	
5B	6		7		8		6		4 1352		13		1352	46.5
5W										1556			1556	
6B	16		18		18		13		13		13 406		406	32.6
6W												841	841	
TOTAL STUDENTS	1496	1415	1498	1619	1405	1295	1579	1251	1352	1556	406	841	15,713	

Cost coefficients printed on form used unless otherwise indicated.

Intermediate Grades Student Assignment Program By Section.

Minimum Transportation Output: 131,782

Description: Increase B 40%

Decrease W 40%

Increase coefficient 4.00

	1		2		3		4		5		6		TOTAL ASSIGNED	ADJUST-MENT
	B	W	B	W	B	W	B	W	B	W	B	W		%
1B	4 1611		7		8		5		6		16		1611	57.1
1W		1213											1213	
2B	7		4 1613		5		6		7		18		1613	53.8
2W				1388									1388	
3B	8		5		4		4 1700		8		18		1700	57.4
3W						188		1073					1261	
4B	8		6		4 1513		5		6		13		1513	62.1
4W						923							923	
5B	6		7		8		6		4 1456		13		1456	52.2
5W										1334			1334	
6B	16		18		18		13		13		13 437		437	37.7
6W												721	721	
TOTAL STUDENTS	1611	1213	1613	1388	1513	1111	1700	1073	1456	1334	437	721	15,170	

Cost coefficients printed on form used unless otherwise indicated.

Intermediate Grades Student Assignment Program By Section.

Minimum Transportation Output: 141,264

Description: Increase B 50%

Decrease W 50%

Increase coefficient 5.00

	1		2		3		4		5		6		TOTAL ASSIGNED	ADJUSTMENT
	B	W	B	W	B	W	B	W	B	W	B	W		%
1B	4 1726		7		8		8		6		16		1726	63.1
1W		1011											1011	
2B	7		4 1728		5		6		7		18		1728	59.9
2W				1157									1157	
3B	6		5		4		4 1821		8		18		1821	61.5
3W						245		895					1140	
4B	8		6		4 1621		5		6		13		1621	70.4
4W						682							682	
5B	6		7		8		6		4 1560		13		1560	58.4
5W										1112			1112	
6B	16		18		18		13		13		13 468		468	43.8
6W												601	601	
TOTAL STUDENTS	1726	1011	1728	1157	1621	927	1821	895	1560	1112	468	601	14,627	

Cost coefficient printed on form used unless otherwise indicated.

Intermediate Grades Student Assignment Program By Section.

Minimum Transportation Output: 149,660

Description: Increase B 60%

Decrease W 60%

Increase coefficient 6.00

	1		2		3		4		5		6		TOTAL ASSIGNED	ADJUSTMENT
	B	W	B	W	B	W	B	W	B	W	B	W		%
1B	4 1841		7		8		8		6		16		1841	69.5
1W		809											809	
2B	7		4 1843		5		6		7		18		1843	66.6
2W				926									926	
3B	8		5		4		4 1942		8		18		1942	65.6
3W						302		717					1019	
4B	8		6		4 1729		5		6		13		1729	79.7
4W						441							441	
5B	6		7		8		6		4 1664		13		1664	65.2
5W										890			890	
6B	16		18		18		13		13		13 499		499	50.9
6W												481	481	
TOTAL STUDENTS	1841	809	1843	926	1729	743	1942	717	1664	890	499	481	14,084	

Cost coefficients printed on form used unless otherwise indicated.

Intermediate Grades Student Assignment Program By Section.

Minimum Transportation Output: 156,970

Description: Increase B 70%

Decrease W 70%

Increase coefficient 7.00.

	1		2		3		4		5		6		TOTAL ASSIGNED	ADJUSTMENT
	B	W	B	W	B	W	B	W	B	W	B	W		%
1B	4 1956		7		8		8		6		16		1956	76.3
1W		607											607	
2B	7		4 1958		5		6		7		18		1958	73.8
2W				695									695	
3B	8		5		4		4 2063		8		18		2063	69.7
3W						359		539					898	
4B	8		6		4 1837		5		6		13		1837	90.2
4W						200							200	
5B	6		7		8		6		4 1768		13		1768	72.6
5W										668			668	
6B	16		18		16		13		13		13 530		530	59.5
6W												361	361	
TOTAL STUDENTS	1956	607	1958	695	1837	559	2063	539	1768	668	530	361	13,541	

Cost coefficients printed on form used unless otherwise indicated.

Table 3

Intermediate Grades Student Assignment Program By Section.

Minimum Transportation Output: 163,194

Description: Increase B 80%

Decrease W 80%

Increase coefficient 8.00

	1		2		3		4		5		6		TOTAL ASSIGNED	ADJUSTMENT
	B	W	B	W	B	W	B	W	B	W	B	W		%
1B	4 2071		7		8		8		6		16		2071	83.6
1W		405											405	
2B	7		4 2073		5		6		7		18		2073	81.7
2W				464									464	
3B	8		5		4		4 2184		8		16		2184	74.8
3W						375		361					736	
4B	8		6		4 1945		5		6		13		1945	100.0
4W													-	
5B	6		7		8		6		4 1872		13		1872	80.8
5W										446			446	
6B	16		18		18		13		13		13 561		561	70.0
6W												241	241	
TOTAL STUDENTS	2071	405	2073	464	1945	375	2184	361	1872	446	561	241	12,998	

Cost coefficients printed on form used unless otherwise indicated.

Intermediate Grades Student Assignment Program By Section.

Minimum Transportation Output: 168,332

Description: Increase B 90%

Decrease W 90 %

Increase coefficient 9.00

	1		2		3		4		5		6		TOTAL ASSIGNED	ADJUSTMENT
	B	W	B	W	B	W	B	W	B	W	B	W		%
1B	4 2186		7		8		8		6		16		2186	91.5
1W		203											203	
2B	7		4 2188		5		6		7		18		2188	90.4
2W				233									233	
3B	8		5		4		4 2305		8		15		2305	86.0
3W						191		183					374	
4B	8		6		4 2053		5		6		13		2053	100.0
4W													-	
5B	6		7		8		6		4 1976		13		1976	89.8
5W										224			224	
6B	16		18		18		13		13		13 592		592	83.0
6W												121	121	
TOTAL STUDENTS	2186	203	2188	233	2053	191	2305	183	1976	224	592	121	12,455	

Cost coefficients printed on form used unless otherwise indicated.

Intermediate Grades Students Assignment Program By Section.

Minimum Transportation Output: 172,304

Description: Increase B 100%
Decrease W 100%
Increase coefficient 10.00

Note: Nominal W Students due to rounding. System all B at this point.

	1		2		3		4		5		6		TOTAL ASSIGNED	ADJUSTMENT
	B	W	B	W	B	W	B	W	B	W	B	W		%
1B	4 2301		7		8		8		6		16		2301	100
1W		1											1	
2B	7		4 2303		5		6		7		18		2303	100
2W				2									2	
3B	8		5		4		4 2426		3		18		2426	100
3W						7		5					12	
4B	8		6		4 2161		5		6		13		2161	100
4W													-	
5B	6		7		8		6		4 2080		15		2080	100
5W										2			2	
6B	16		18		16		15		13		13 623		623	100
6W												1	1	
TOTAL STUDENTS	2301	1	2303	2	2161	7	2426	5	2080	2	623	1	11,912	

Cost coefficients printed on form used unless otherwise indicated.

Table 40

Optimal Program With Wide Racial Constraints

	1		2		3		4		5		6		DUMMY	TOTAL
	B	W	B	W	B	W	B	W	B	W	B	W		
1B	4 1151		7		8		8		6		16			1151
1W		2021												2021
2B	7		4 1153		5		6		7		18			1153
2W				2312										2312
3B	8		5		4		4 1159		8		18			1159
3W						20		1785						1805
4B	8		5		4 1081		5 57		6		13			1138
4W						1827								1827
5B	6		7		8		6		4 1040		13			1040
5W										2222				2222
6B	10		18		18		13		13		13 313			313
6W												1201		1201
Total Units	1151	2021	1153	2312	1081	1847	1216	1785	1040	2222	313	1201		17343

Intermediate Grades Students Assignment Program By Section.

Percent of Black Students = 50%

Minimum = 83,034

	1		2		3		4		5		6		DUMNY	BLDG. CAPACITY
	B	W	B	W	B	W	B	W	B	W	B	W		TOTAL
1B	4 1151	487	7	-	8		8		6		16		0	1638
1W		1534											104	1638
2B	7		4 1153	625	5		6		7		18			1778
2W				1687									92	1779
3B	8		5		4		4 1216	264	8		18			1480
3W								1481						1481
4B	8		6		4 1081	475	5		6		13			1556
4W						1372		40					144	1556
5B	6		7		8		6		4	1634	13			1634
5W									1040	588		6		1634
6B	16		18		18		13		13		13	775		775
6W											313	420		775
TOTAL STUDENTS	1151	2021	1153	2312	1081	1847	1216	1785	1040	2222	313	1201		17,724

Intermediate Grades Students Assignment Program By Section.

Percent of Black Students = 45%

Minimum = 83,034

	1		2		3		4		5		6		DUMMY	BLDG. CAPACITY
	B	W	B	W	B	W	B	W	B	W	B	W		TOTAL
1B	4 1151	323.2	7		8		8		6		16		0	1474.2
1W		1697.8											104	1801.8
2B	7		4 1153	447.1	5		6		7		18			1600.1
2W				1864.9									92	1956.9
3B	8		5		4		4 1216	115.9	8		18			1331.9
3W								1629.1						1629.1
4B	8		6		4 1081	319.4	5		6		13			1400.4
4W						1527.6		40					144	1711.6
5B	6		7		8		6		4	1470.6	13			1470.6
5W									1040	751.4		6		1797.4
6B	16		18		18		13		13		13	697.5		697.5
6W											313	497.5	42	852.5
TOTAL STUDENTS	1151	2021	1153	2312	1081	1847	1216	1785	1040	2222	313	1201	382	17724

Table 43

Intermediate Grades Students Assignment Program by Section.

Percent of Black Students = 40%

Minimum = 83,034

	1		2		3		4		5		6		DUMNY	Bldg. CAPACITY
	B	W	B	W	B	W	B	W	B	W	B	W		TOTAL
1B	4 1151	159.4	7		8		8		6		16		0	1310.4
1W		1861.6											104	1965.6
2B	7		4 1153	259.2	5		6		7		18			1422.2
2W				2042.8									92	2134.8
3B	8		5		4		4 1183.8		8		18			1183.8
3W							32.2	1745						1777.2
4B	8		6		4 1081	163.8	5		6		13			1244.8
4W						1683.2		40					144	1867.2
5B	6		7		8		6		4	1307.2	13			1307.2
5W									1040	914.8		6		1960.8
6B	16		18		18		13		13		13	620		620.0
6W											313	575	42	930.0
TOTAL STUDENTS	1151	2021	1153	2312	1081	1847	1216	1785	1040	2222	313	1201	382	17,724

Intermediate Grades Students Assignment Program By Section.

Percent of Black Students = 35%

Minimum = 83,034

	1		2		3		4		5		6		DUMMY	BLDG. CAPACITY
	B	W	B	W	B	W	B	W	B	W	B	W		TOTAL
1B	4 1146.6		7		8		8		6		16		0	1146.6
1W	4.4	2021											104	2129.4
2B	7		4 1153	91.3	5		6		7		18			1244.3
2W				2220.7									92	2312.7
3B	8		5		4		4 1035.7		8		18			1035.7
3W							180.3	1745						1925.3
4B	8		6		4 1081	8.2	5		6		13			1089.2
4W						1838.8		40					144	2022.8
5B	6		7		8		6		4 1143.8		13			1143.8
5W									1040	1078.2		6		2124.2
6B	16		18		18		13		13		13	542.5		542.5
6W											313	652.5	42	1007.5
TOTAL STUDENTS	1151	2021	1153	2312	1081	1847	1216	1785	1040	2222	313	1201	382	17,724

Intermediate Grades Students Assignment Program By Section.

Percent of black Students = 30%

Minimum = 83.034

	1		2		3		4		5		6		DUMMY	BLDG. CAPACITY
	B	W	B	W	B	W	B	W	B	W	B	W		TOTAL
1B	4 982.8		7		8		8		6		16		0	982.8
1W	168.2	2021											104	2293.2
2B	7		4 1066.4		5		6		7		18			1066.4
2W			86.6	2312									92	2498.6
3B	8		5		4		4 887.6		8		18			887.6
3W							328.4	1745						2073.4
4B	8		6		4 933.6		5		6		13			933.6
4W					147.4	1847		40					144	2178.4
5B	6		7		8		6		4	980.4	13			980.4
5W									1040	1241.6		6		2287.6
6B	16		18		18		13		13		13	465		465.0
6W											313	730	42	1085.0
TOTAL STUDENTS	1151	2021	1153	2312	1081	1847	1216	1785	1040	2222	313	1201	382	17,724

Intermediate Grades Students Assignment Program By Section.

Percent of Black Students = 25%

Minimum = 83,034

	1		2		3		4		5		6		D U M M Y	BLDG. CAPACITY
	B	W	B	W	B	W	B	W	B	W	B	W		TOTAL
1B	4 819		7		8		8		6		16		0	819.0
1W	332	2021											104	2457.0
2B	7		4 888.5		5		6		7		18			888.5
2W			264.5	2312									92	2668.5
3B	8		5		4		4 739.5		8		18			739.5
3W							474.5	1745						2221.5
4B	8		6		4 778		5		6		13			778.0
4W					303	1847		40					144	2334.0
5B	6		7		8		6		4	817	13			817.0
5W									1040	1405		6		2451.0
6B	16		18		18		13		13		13	387.5		387.5
6W											313	807.5	42	1162.5
TOTAL STUDENTS	1151	2021	1153	2312	1081	1847	1216	1785	1040	2222	313	1201	382	17,724

Table 47

Intermediate Grades Students Assignment Program By Section

Percent of Black Students = 20%

Minimum = 83,034

	1		2		3		4		5		6		DUMMY	BLDG. CAPACITY
	B	W	B	W	B	W	B	W	B	W	B	W		TOTAL
1B	4	655.2	7		8		8		6		16		0	655.2
1W	1151.0	1365.8											104	2620.8
2B	7		4		5		6		7		18		92	710.6
			618.6											
2W			534.4	2312.0										2846.4
3B	8		5		4		4		8		18			
							591.4							591.4
3W							624.6	1745.0						2369.6
4B	8		6		4		5		6		13			
					622.4									622.4
4W					458.6	1847.0		40.0					144	2489.6
5B	6		7		8		6		4		13			
									653.6					653.6
5W									386.4	2222		6.0		2614.4
6B	16		18		18		13		13		13			
												310.0		310.0
6W											313.0	885.0	42	1240.0
TOTAL STUDENTS	1151	2021	1153	2312	1081	1847	1216	1785	1040	2222	313	1201	382	17,724

Table 43

Intermediate Grades Students Assignment Program By Section.

Percent of Black Students = 15%

Minimum = 85 934

	1		2		3		4		5		6		DUMMY	BLDG. CAPACITY
	B	W	B	W	B	W	B	W	B	W	B	W		TOTAL
1B	4 491.4		7		8		8		6		16		0	491.4
1W	1151	1529.6											104	2784.6
2B	7		4 440.7		5		6		7		18		92	532.7
2W			732.3	2312										3024.3
3B	8		5		4		4 443.3		8		18			443.3
3W							772.7	1745.0						2517.7
4B	8		6		4 466.8		5		6		13			466.8
4W					614.2	1847.0		40.0					144	2645.2
5B	6		7		8		6		4 490.2		13			490.2
5W									549.8	2222		6.0		2777.8
6B	16		18		18		13		13		13	232.5		232.5
6W											313.0	962.5	42.0	1317.5
TOTAL STUDENTS	1151	2021	1153	2312	1081	1847	1216	1785	1040	2222	313	1201	382	17,724

Table 49

Intermediate Grades Students Assignment Program By Section.

Percent of Black Students = 10%

Minimum = 83,034

	1		2		3		4		5		6		DUMMY	BLDG. CAPACITY TOTAL
	B	W	B	W	B	W	B	W	B	W	B	W		
1B	4	327.6	7		8		8		6		16		0	327.6
1W	1151	1693.4											104	2948.4
2B	7		4	262.8	5		6		7		18		92	354.8
2W			890.2	2312.0										3202.2
3B	8		5		4		4	295.2	8		18			295.2
3W							920.8	1745.0						2665.8
4B	8		6		4	311.2	5		6		13			311.2
4W					769.8	1847.0		40.0					144	2800.8
5B	6		7		8		6		4	326.8	13			326.8
5W									713.2	2222.0		6.0		2941.2
6B	16		18		18		13		13		13	155.0		155.0
6W											313.0	1040.0	42	1395.0
TOTAL STUDENTS	1151	2021	1153	2312	1081	1847	1216	1785	1040	2222	313	1201	382	17,724

Table 30

Intermediate Grades Students Assignment Program by Section.

Percent of Black Students = 5%

Minimum = 83,034

	1		2		3		4		5		6		BUMMY	BLDG. CAPACITY
	B	W	B	W	B	W	B	W	B	W	B	W		TOTAL
1B	4	163.8	7		8		8		6		16		0	163.8
1W	1151	1857.2											104	3112.2
2B	7		4		5		6		7		18		92	176.9
2W			84.9											
			1068.1	2312										3380.1
3B	8		5		4		4		8		18			
							147.1							147.1
3W							1068.9	1745						2813.9
4B	8		6		4		5		6		13			
					155.6									155.6
4W					925.4	1847.0		40.0					144	2956.4
5B	6		7		8		6		4		13			
									163.4					163.4
5W									876.6	2222		6.0		3104.6
6B	16		18		18		13		13		13			
												77.5		77.5
6W											313.0	1117.5	42	1472.5
TOTAL STUDENTS	1151	2021	1153	2312	1081	1847	1216	1785	1040	2222	313	1201	382	17,724

THE FINAL STUDY

RICHMOND

[illegible]

		[illegible]	[illegible]	[illegible]
[illegible]	1 [illegible]	[illegible]	[illegible]	[illegible]
	2 [illegible]	[illegible]	[illegible]	[illegible]
	3 [illegible]	[illegible]	[illegible]	[illegible]
	4 [illegible]	[illegible]	[illegible]	[illegible]
	5 [illegible]	[illegible]	[illegible]	[illegible]
	6 [illegible]	[illegible]	[illegible]	[illegible]
	7 [illegible]	[illegible]	[illegible]	[illegible]
	8 [illegible]	[illegible]	[illegible]	[illegible]
	9 [illegible]	[illegible]	[illegible]	[illegible]
	10 [illegible]	[illegible]	[illegible]	[illegible]

SCHOOL KEY

I. Chandler	III. Binford	V. Norby	VII. East End	IX. Bainbridge
II. Hill	IV. Graves	VI. Thompson	VIII. Westover	X. Elkhardt

SCHOOL / SECTION	I	II	III	IV	V	VI	VII	VIII	IX	X
3G	3½	3 3/4	3 7/8	4	6	9	7	6	6	10½
3H	2 3/4	4½	4½	3 5/8	6 3/4	9 7/8	6 5/8	6 5/8	6 5/8	9 5/8
4B	6½	3½	4 7/8	6	7 5/8	6½	8 7/8	7	7½	10
4C	7½	2½	4	5½	7	6	8½	6	6 3/4	11 1/8
4F	2 7/8	2¼	3 3/4	4	5 5/8	8 3/8	6 3/4	5½	5 3/4	9½
4G	2 3/4	2 7/8	3	3 5/8	5 5/8	8 3/8	5¼	5 3/8	6 1/8	9½
4H	3¼	3 5/8	3 3/4	2 7/8	5	9	5 7/8	6 3/8	5	9½
4J	1¼	3 5/8	3 3/4	3 7/8	5½	9 1/8	6	6¼	5	9½
4K	1½	5	3 5/8	3 3/8	4 3/8	10½	6 5/8	7½	5½	9 5/8
5B	7	3 1/8	4½	6	7 3/4	4½	8½	6¼	7	9
5C	6½	2¼	3 3/4	5	6 3/8	5½	7 5/8	5 3/4	6 5/8	9¼
5D	5½	1 5/8	3¼	4¼	5 7/8	6 5/8	5 7/8	5 3/8	6	10 3/8
5E	3 5/8	1 3/8	3 1/8	3½	5	7 3/4	6	5	5	8½
5F	3¼	1	2 3/8	2 7/8	4½	7¼	5½	4	4½	8
5G	1 3/4	2	1 7/8	2 5/8	4¼	7½	5 1/8	4 5/8	4¼	8½
5H	1¼	3½	2¼	2	4	8 3/4	5	5 3/4	4	8¼
5J	½	4 3/8	3	2 5/8	4 7/8	5 7/8	5 5/8	6½	4 3/4	9
5K	1 1/8	5	4	3 5/8	3¼	10½	5	7 3/4	6	10 1/8
6B	6 7/8	3	4	6¼	7 3/4	5¼	8½	6	6½	8 5/8
6C	5 7/8	2 5/8	3½	5	6½	5 5/8	7½	5¼	6	9 3/8
6D	4 7/8	1½	2 3/4	4 1/8	5 3/4	6 3/8	6 3/4	4 3/4	5 3/8	8 3/8
6E	4¼	½	2¼	3 3/8	5	7¼	6	4	4 7/8	8
6F	3	5/8	1 5/8	2¼	3 7/8	8	4 7/8	3 3/4	4	7½
6G	2½	2¼	1¼	2	3 5/8	7 5/8	4½	4 5/8	3½	8 5/8
6H	1½	3	1 3/4	1½	3¼	8½	4	5½	3 3/8	7½

	I	II	III	IV	V	VI	VII	VIII	IX	X
6J	1	3 3/4	2½	2	2 3/4	9	4/7/8	6 5/8	4	8 3/8
6K	1¼	4 7/8	3½	3	2 7/8	10 3/8	4 1/8	7 3/8	5 1/8	9 3/8
6L	1¼	4 7/8	4 1/8	2 7/8	3½	10 5/8	5	7½	5 3/4	10
7EE	11 3/4	7 3/4	8 3/4	10 3/4	12 3/8	4 5/8	13 3/8	7	9 5/8	8 7/8
7DD	11	7 3/8	8¼	10 3/8	11 7/8	3½	12 3/4	6 3/8	9½	9
7CC	10 7/8	7 1/8	8	10	11 5/8	4 3/8	12½	6½	8 3/	8
7BB	9 3/8	5 5/8	6½	8	10 1/8	3 1/8	10 5/8	4 5/8	7¼	6 7/8
7A	9	5 3/8	6½	7 3/4	9 3/8	3 3/8	10 3/8	4 7/8	7½	6 7/8
7B	9 3/4	5 3/4	6 5/8	8	9 7/8	2½	10 3/8	3 3/8	6	8
7C	9	6¼	6 5/8	8 3/4	10 7/8	2 1/8	11 1/4	3	4½	6
7D	5	1 3/4	2 5/8	4½	6¼	7 1/8	7 1/8	4 3/8	5	8½
7E	4 3/8	1½	1½	3½	5¼	6¼	6	3½	4	7½
7F	3 3/4	1	3/4	2½	4	6	4 7/8	3	3½	6 7/8
7G	2 3/4	1½	5/8	1¼	3	7 1/8	3 7/8	4¼	2 3/4	7
7H	2½	2 3/4	1 3/8	½	2 3/8	8 1/8	3½	5 1/8	2½	6 3/4
7J	2¼	3 3/4	2 3/8	7/8	1½	9¼	3 3/8	6¼	2 7/8	7
7K	3¼	5½	4¼	3½	1½	11	3¼	7½	4 5/8	8 7/8
7L	4	6	4 7/8	3½	1 5/8	10	3 1/8	7	4 5/8	8 7/8
7M	4 1/8	6¼	5	3 3/4	1 5/8	10	3 1/8	7	4 7/8	9 1/8
8EE	12¼	8½	9 5/8	10 7/8	12½	3 3/4	12 7/8	6 7/8	9½	8 3/4
8DD	11¼	7½	9 3/8	6¼	12	3	12 3/4	6¼	8 5/8	8 1/8
8CC	10 3/8	6 3/4	7 5/8	9	10 3/4	2 1/8	11 3/4	5½	8	7 3/8
8BB	9 3/8	5 3/4	6 5/8	8	9 3/4	2 1/8	11	4 3/8	7	6¼
8A	8½	6 1/8	6 7/8	8	8½	3/4	9½	3 5/8	6	5½
8B	8 5/8	6	6	7	7½	1 5/8	8½	2 5/8	5	5½

	I	II	III	IV	V	VI	VII	VIII	IX	X
8C	8 1/8	5½	5 5/8	6 5/8	7½	1 3/4	8 1/8	2 1/8	4 5/8	5½
8D	5	2 3/8	2½	3 7/8	5½	5 7/8	6 5/8	2 3/4	4 7/8	6 3/4
8E	4 3/4	2	1 7/8	3 5/8	5¼	5 1/8	6 1/8	2¼	4¼	6¼
8F	4 1/8	2 1/8	3/4	2½	4	6 5/8	5	3 3/4	3½	7½
8G	3 3/8	2¼	½	1½	3 1/8	9 3/8	4	3½	2¼	6 3/8
8H	3 1/8	2 7/8	1 1/8	½	2	7	5	8	1 7/8	6
8J	3¼	3½	2½	5/8	1½	7½	2 1/8	4½	2	6 3/8
8K	4	4 3/4	3½	1 7/8	1/16	8 3/8	1 3/4	5½	3½	7 7/8
8L	1 3/4	5½	4 3/8	3	1	9½	1 3/4	6¼	4	8 3/8
8M	5 3/8	6	5	3 3/8	1 5/8	10	1 3/4	7	4 3/4	9
9EE	11 1/8	7 3/4	8 3/4	10¼	11 7/8	3	12 3/4	6¼	8 3/4	8 1/8
9DD	10 3/4	7½	8¼	9 3/4	11½	2½	12 3/8	5 3/4	8¼	7 5/8
9CC	10½	6 7/8	7 7/8	9¼	10 7/8	2	11 7/8	5½	7 3/4	7¼
9A	9½	6 3/4	6 5/8	7 5/8	9	1/8	9 3/8	3 3/8	5 7/8	5¼
9B	8 3/4	6 1/8	6	7	7 3/4	1 5/8	8 5/8	2 5/8	5¼	4
9C	7 7/8	5¼	5 1/8	6½	6 7/8	3¼	7 3/4	1 3/4	4 3/8	6
9D	7 3/4	4 3/8	4¼	5¼	6½	2½	7½	1	3 5/8	5¼
9E	5½	2 7/8	2 3/4	4 3/4	6	3 3/4	6 7/8	7/8	3	4 3/4
9F	5	2 7/8	1 5/8	3	4 3/4	7	5 5/8	4 1/8	3 3/4	8
9H	4 5/8	4½	2 3/4	2	2½	5 7/8	3 3/8	3	5⅛	5
9J	4¼	3 7/8	2 1/8	1 5/8	1 1/8	7¼	2	4¼	1½	5 5/8
9K	4½	4 3/8	3	1 7/8	1 1/8	8	1	5 1/8	1 7/8	6 3/4
9L	5¼	5½	4¼	2 7/8	1¼	9 1/8	½	6 1/8	4	7 5/8
9M	5 7/8	6 3/8	5	3 5/8	1 7/8	10	1½	7	4 3/4	8½
10B	8¼	5½	5½	6 5/8	7¼	2 7/8	8¼	2	4 3/4	3 7/8

	I	II	III	IV	V	VI	VII	VIII	IX	X
10C	7½	4 7/8	4 3/4	5 7/8	6 ½	3	7½	1¼	4	3 3/4
10D	6 7/8	4¼	4 1/8	5	5 7/8	3 7/8	6 3/4	½	3½	5
10E	6	3½	3 3/8	4	5	3 5/8	5 3/4	3/8	2¼	4½
10F	6 3/4	4 1/8	4	3 3/4	4½	4	5½	1	1 3/4	4 3/4
10G	4 7/8	4 3/8	2 3/4	2½	3 1/8	5 1/8	4	2 1/8	5/8	5
10H	5	4 7/8	3	2 3/4	2½	2 1/8	3 3/8	3 1/8	3/8	4 3/8
10J	4 3/8	4 7/8	3 1/8	2½	2 3/8	6 3/4	3¼	4	1 3/8	5 3/8
10K	4 3/4	5½	3½	2 7/8	2¼	8 5/8	1 3/8	5 5/8	3 3/8	7 1/8
10L	5 1/8	5 5/8	3 7/8	3 1/4	2 5/8	9	7/8	6	3 5/8	7 5/8
10M	5½	6½	5	3 5/8	2 3/4	9 7/8	1 1/8	6 7/8	4 3/8	8¼
11B	8½	5 7/8	5 3/4	6 3/4	8 3/4	3	8 3/8	2¼	5	3 7/8
11C	8 3/8	5 3/4	5 5/8	6 3/4	7½	4	8 3/8	2 1/8	4½	4¼
11D	8	5 1/8	5	5½	5 3/4	4 7/8	6 3/4	2	3¼	3 5/8
11E	5 7/8	4¼	4 1/8	4 5/8	4 7/8	4 1/8	5 3/4	1 3/8	2¼	3¼
11F	6½	5 1/8	5	4	4¼	4 5/8	5 1/8	1 7/8	2 5/8	3
11G	5 3/4	5¼	5¼	4	3 5/8	5 1/8	4 5/8	2¼	1	3 3/8
11H	5 5/8	5 3/8	3½	3¼	4	6½	4 3/8	3½	7/8	5
11J	5½	5 7/8	4	3½	3¼	7 3/8	4 3/4	4 1/8	1 5/8	5 5/8
11L	6	6¼	4½	3 7/8	3	9 5/8	1 5/8	6 3/4	4 1/8	8¼
11M	6 1/8	6 5/8	4 7/8	4¼	3 3/8	10	1 3/8	7	4½	8 5/8
12B	8 7/8	6¼	6 1/8	6 5/8	6 7/8	3¼	7 7/8	3	4¼	2 1/8
12C	8 7/8	6¼	6 1/8	6 5/8	6 7/8	3 7/8	7 7/8	3	4¼	2 3/4
12D	8 3/8	1 3/4	5 5/8	5 7/8	6¼	4 3/4	7 1/8	3 3/4	3 5/8	1 3/4
12E	7½	6 1/8	6	5 1/8	5½	6 7/8	6 3/8	3	2 3/4	2 7/8
12F	6 7/8	5½	5½	4 3/8	3 5/8	5½	5½	2½	2 1/8	2½

	I	II	III	IV	V	VI	VII	VIII	IX	X
12G	6 5/8	6½	4½	4½	4½	6 1/8	5½	3	2	3½
12H	6½	5 7/8	4 1/8	3 7/8	4 1/8	7	5	4 1/8	1½	5½
12J	6	6 3/4	4 3/4	4 1/8	4	8	5	5 1/8	2 3/8	6½
13C	9½	8 1/8	8	7 1/8	7½	5	8 3/4	4 7/8	4 7/8	5/8
13D	8½	7 1/8	7	6 1/8	6½	6 1/8	7 3/8	3 3/4	3 3/4	1
13E	8	6 5/8	6	5 5/8	5 7/8	5 3/4	6 3/4	3½	3¼	1½
13F	7½	6¼	5 3/8	5 1/8	5 3/8	6 1/8	6½	3	2 3/4	2 5/8
13G	7 3/4	6 3/4	5 5/8	5 3/8	5 5/8	6 5/8	6½	3½	3	4
13H	7	6½	4 7/8	4½	4 3/4	7 3/4	5 3/8	4 3/4	2¼	4½
14C	9 5/8	8¼	7½	7¼	7½	5 1/8	8¼	4 7/8	4 7/8	5/8
14D	9¼	7 7/8	7 1/8	6 7/8	7 1/8	6	8	4½	4 5/8	7/8
14D	8 3/4	7½	6 5/8	6 3/8	6½	6½	7½	4½	4	1 3/8
14F	8 5/8	7½	6½	6½	6½	7	7 3/8	4 1/8	4	2½
14G	8½	7½	6 3/8	6	6 3/8	7 7/8	7¼	3 7/8	3 3/4	3 3/4
14H	7 3/4	7½	5 5/8	5 3/8	5 5/8	8 5/8	6½	5	3	4½
14J	8 3/8	8	6¼	6	6¼	9¼	7 1/8	5½	3 5/8	5 1/8
15D	9 5/8	8¼	7 ½	7¼	7½	6	8¼	4 7/8	5	1¼
15E	9¼	8	7¼	7	7¼	7 1/8	8	4 3/4	4 5/8	2 3/4
15F	9 5/8	8 3/8	7½	7¼	7½	7 7/8	8 3/8	5	4 7/8	3
15G	9 3/4	8 7/8	7 5/8	7 3/8	7 5/8	8¼	8½	4 3/4	5	3½
15H	8½	8	6½	6 1/8	6½	9½	7 3/8	6	3 7/8	4 5/8

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School 2 - Binford

Coordinate	Number of Students	Black	White
135	40		X
136	20		X
136	2	X	
141	40		X
144	2	X	
145	69		X
146	3	X	
	Total 178		

School 3 - Chandler

Coordinate	Number of Students	Black	White
259	6		X
259	3	X	
260	5	1	X
268	18	X	
272	103		X
283	67		X
283	65	X	
298	7		X
298	197	X	
299	5		X
299	391	X	
313	69	X	
Total	926		

School 4 - East Mad

Coordinate	Number of Students	Black	White
391	1		X
391	93	X	
406	5		X
406	198	X	
407	1		X
407	306	X	
422	73		X
422	32	X	
423	3		X
423	13	X	
438	62	X	
Total	837		

School 5 - Graves

Coordinate		Number of Students	Black	White
546		1		X
546		554	X	
547		251	X	
	Total	806		

School 6 - Mosby

Coordinate	Number of Students	Black	White
613	35		X
614	12		X
614	1	X	
620	3		X
621	33		X
621	3	X	
622	32		X
622	22	X	
627	9		X
627	330	X	
630	1		X
631	21		X
640	20		X
641	1		X
642	20		X
643	14		X
644	11		X
645	1		X
647	24		X
647	47	X	
648	4		X
648	12	X	
656	47		X

School 6 - Mosby (continued)

Coordinate	Number of Students	Black	White
657	20		X
658	6		X
659	34		X
659	1	X	
660	38		X
660	11	X	
661	30		X
663	13		X
663	31	X	
672	27		X
673	6		X
674	10		X
674	2	X	
675	6		X
675	7	X	
676	16		X
676	3	X	
678	8		X
679	11		X
685	9		X
686	21		X
691	108		X
691	324	X	
697	49		X
698	65		X

School 6 - Nosby (continued)

Coordinate	Number of Students	Black	White
698	3	X	
700	24		X
700	25	X	
701	10		X
701	112	X	
707	17		X
707	13	X	
Total	187		

School Y - Thompson

Coordinate	Number of Students	Black	White
804	223	X	
805	2		X
805	167	X	
816	25		X
816	113	X	
817	30		X
Total	931		

School 8 - Westover Hills

Coordinate	Number of Students	Black	White
802	135	X	
906	1		X
931	91		X
932	50		X
933	35		X
933	1	X	
934	74		X
934	67	X	
943	54		X
955	47		X
955	1	X	
962	12		X
962	4	X	
Total	572		

School 9 - Hill

Coordinate	Number of Students	Black	White
1046	12		X
1046	10	X	
1047	6	X	
1048	86	X	
1050	55	X	
1068	18		X
1068	251	X	
1069	57		X
1069	96	X	
1078	67		X
1078	7	X	
Total	665		

School 10 - Elkhardt

Coordinate	Number of Students	Black	White
1196	38		X
1197	55		X
1197	2	X	
1198	30		X
1202	37		X
1203	88		X
1203	2	X	
1204	49		X
1205	82		X
1205	3	X	
1207	85		X
1207	9	X	
1209	25		X
1210	36		X
1210	1	X	
1211	51		X
1212	61		X
1212	1	X	
1213	36	X	Y
1213	13	X	
1214	20	X	X
1216	9		X
1216	1	X	
1217	21		X
1217	2	X	
1218	28		X

School 10 - Elkhardt (continued)

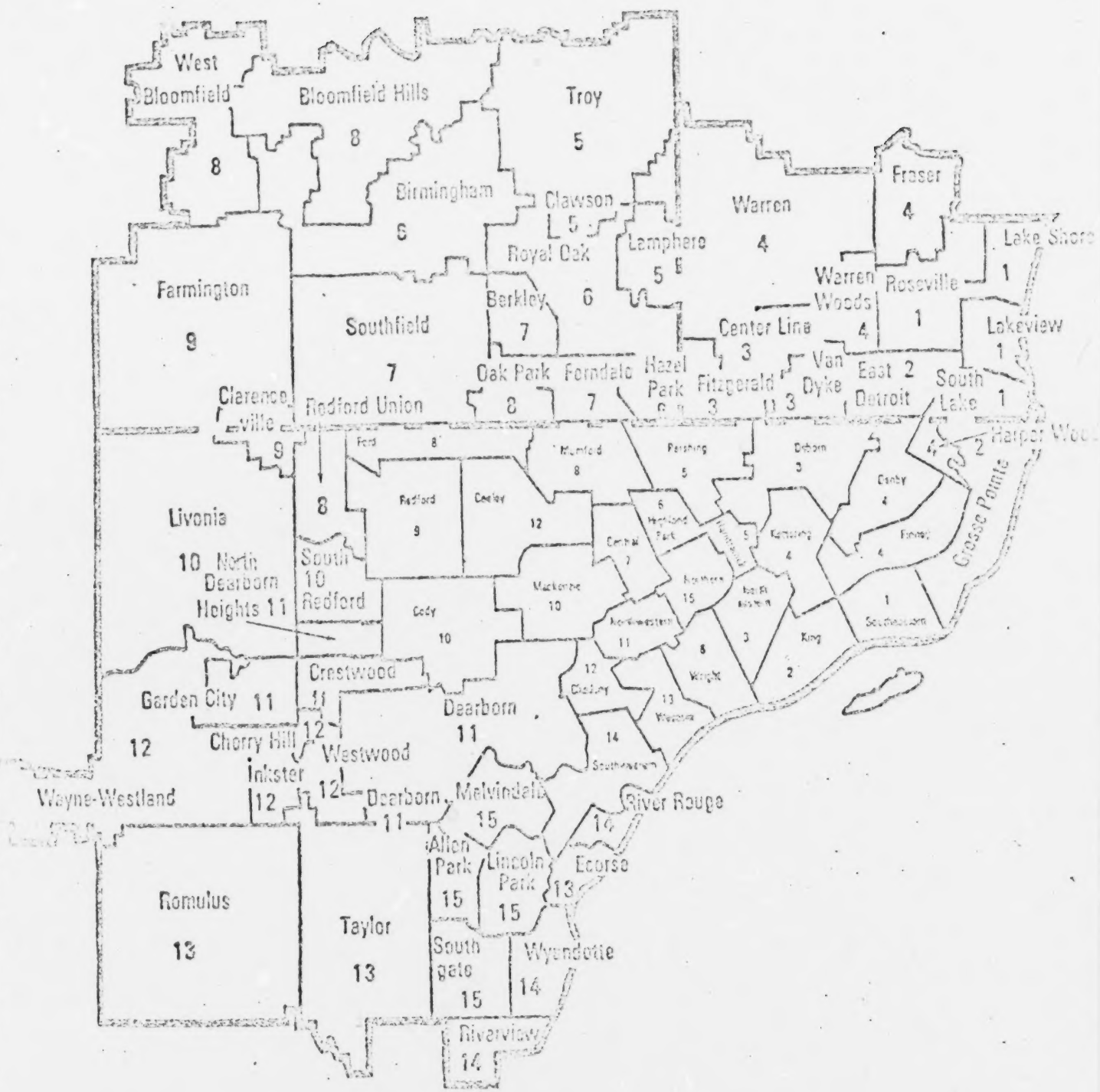
Coordinate	Number of Students	Black	White
1218	1	X	
1219	22		X
1219	2	X	
Total	610		

Districts in the Detroit planning area for consolidation were paired in order to assure a racial balance which would better reflect the area's actual racial composition.

The following districts were considered:

1. Southeastern
2. King
3. North Eastern
4. Finney
5. Pershing
6. Wright
7. Central
8. Ford
9. Redford
10. Cody, MacKenzie
11. Northwestern
12. Chadsey
13. Western
14. Southwestern
15. Northern
16. Wyandotte, Riverview
17. Southgate, Lincoln Park, Allen Park, Melvindale
18. Ecorse
19. River Rouge
20. Dearborn, Crestwood, Garden City
21. Cherry Hill, Inkster, Wayne-Westland
22. Livonia
23. Farmington
24. Southfield, Berkeley
25. Royal Oak, Birmingham, Hazel Park
26. West Bloomfield, Bloomfield Hills
27. Troy, Lamphere
28. Warren, Fraser
29. Lake Shore, Roseville, Lakeview, South Lake
30. East Detroit
31. Van Dyke, Center Line, Fitzgerald
32. Oak Park
33. Romulus, Taylor
34. Harper Wood

Detroit Study Area



The same numerical identification had been used in the table containing the Detroit technical coefficients.

Approximate student population at high schools had been compiled from the statistical reports of the Detroit School Board.

	B	W		B	W
1.	12,708	2,636	22.	8	37,807
2.	9,496	282	23.	9	20,118
3.	11,860	10,265	24.	814	41,696
4.	25,858	17,902	25.	6,166	46,000
5.	10,008	4,563	26.	0	14,081
6.	6,359	1,551	27.	0	16,732
7.	12,702	21	28.	53	45,903
8.	14,854	7,862	29.	261	36,922
9.	683		30.	6	12,689
10.	22,617	13,826	31.	5	19,221
11.	14,265	30	32.	590	5,260
12.	17,824	6,719	33.	1,264	23,807
13.	4,964	4,409	34.	0	1,975
14.	4,836	5,395	TOTAL	196,930	544,913
15.	8,877	39	Total Black		196,930
16.	1	11,959	Total White		544,913
17.	11	29,101	Student Population:		741,843
18.	2,268	1,811			
19.	1,723	2,069			
20.	1,842	59,641			
21.	3,998	27,938			

The student population in each district was considered as school capacity for the same district. These figures were then subdivided into white and black subcapacities according to the average racial composition of the area.

	B	W		B	W
1.	3,860	11,484	22.	9,521	28,294
2.	2,462	7,316	23	5,068	15,059
3.	5,571	16,554	24.	10,704	31,806
4.	11,018	32,742	25.	13,135	39,031
5.	3,669	10,902	26.	3,545	10,536
6.	1,991	5,919	27.	4,213	12,519
7.	3,203	9,520	28.	11,571	34,385
8.	5,719	16,997	29.	9,362	27,821
9.	3,869	11,497	30.	3,196	9,499
10.	9,176	27,267	31.	4,841	14,385
11.	3,599	10,696	32.	1,473	4,377
12.	6,180	18,363	33.	6,312	18,759
13.	2,360	7,013	34.	497	1,478
14.	2,576	7,655	Total	186,778	555,065
15.	2,245	6,671			
16.	3,011	8,949			
17.	7,330	21,782			
18.	1,027	3,052	Total Black		186,778
19.	954	2,838	Total White		555,065
20.	15,481	46,002	Student Population:		741,843
21.	8,039	23,897			

Technical coefficients, distances between district centers, were calculated for each pair of districts and are shown in the following table.

Technical Coefficients, Detroit

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	0	2	2.5	1.5	4.5	3.5	5	7.5	9	7.5	5	7.5	4.5	6.5	3.5
2		0	1	2.5	4.5	2	4	7	8	6	3	6	2.5	4.5	2.5
3			0	2.5	3	1	2.5	5.5	6.5	4.5	2	5	2.5	4.5	1
4				0	3.5	4	4.5	7	8.5	7	4.5	6	5	7	3.5
5					0	4	2	3.5	5.5	4.5	3.5	4.5	4.5	6.5	2.5
6						0	2.5	5.5	6	4	1.5	4.5	1	3	1.5
7							0	3	4	3	1.5	3	3	4.5	1.5
8								0	2.5	3.5	4	5	5.5	6.5	4.5
9									0	2.5	4.5	2	6	6	5.5
10										0	2.5	2.5	3.5	4	3.5
11											0	3	1	3	1.5
12												0	1.5	2	3
13													0	2	2.5
14														0	4
15															0

	A	B	C	D	E	F	G	H	L	J	K	L	M	N	O	P	R	S	T
1	10	9	8	6.5	9	14	13	14	10	8.5	14.5	10.5	7	5.5	4	5	8	13.5	2.5
2	8.5	7	6	5	7.5	12	11.5	13	9	8.5	14	10.5	8	7	5.5	5.5	7.5	11.5	4.5
3	8.5	7	6	4.5	6.5	11.5	10	11.5	8	7.5	12.5	9.5	7.5	7	5	4.5	6	11	5
4	11	9.5	8.5	7.5	9	14	12.5	13	9	7.5	13	9	5.5	4.5	3	3.5	7	13.5	2
5	11	9	8	7	7	11.5	9.5	9.5	5.5	4.5	10	6.5	4.5	6	4	2	3.5	12	5.5
6	7.5	6	4.5	3.5	5.5	10.5	9.5	11.5	8	8	13	10	8.5	8	6.5	5.5	6.5	10	6
7	9	7	6	5.5	5.5	9.5	8	9	5.5	5.5	10.5	7.5	6.5	8	6	4	4	10	7
8	10.5	8.5	8.5	7.5	6	9	6	6.5	2.5	3	7.5	5.5	6.5	9	7	5	1	10.5	9
9	9.5	7	7.5	7	4	6.5	4	5.5	3	5.5	8	7.5	9	11.5	9.5	7	3	8	11
10	7.5	5	5	4.5	3	7	5.5	7.5	5	6.5	10.5	9	9	10.5	8.5	6.5	4	7.5	9.5
11	7.5	5.5	5	4	4.5	9.5	8	10	6.5	7	11.5	9	8	8.5	6.5	5.5	5	9	7
12	6.5	4	3.5	2.5	3.5	8.5	7.5	10	7	7.5	12.5	10	9.5	10	8	6.5	5.5	8	8
13	6.5	4.5	3.5	2.5	4.5	9.5	9	11	8	8	13	10.5	9	9	7.5	6.5	6.5	8	7
14	4.5	2.5	2	1	3.5	8	8.5	11	8.5	9.5	14	12	11	11	9.5	8	7.5	7	9
15	8.5	7	6	5	6	10.5	9	10.5	6.5	6	11.5	8.5	6.5	7	5	4	5	10.5	5.5

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	R	S	T
16A	0	2.5	2.5	4	5.5	8	10.5	14	12.5	14	17.5	16	15.5	15.5	13.5	12.5	11.5	5.5	15.5
17B		0	1.5	2.5	3	6.5	8	11.5	10	11.5	15	14	13.5	14	12	10.5	9	4.5	11.5
18C			0	1	4	8	9	12.5	10	11	15.5	13.5	12.5	12.5	11	10	9	6	10.5
19D				0	4	8.5	9	12	9.5	10.5	15	12.5	11.5	11.5	10	9	8.5	7	9
20E					0	5	5	8.5	7	9	12	11.5	12	13	11	9.5	6.5	4.5	11.5
21F						0	4	8	9	12	13	14	15.5	17.5	15.5	13	9.5	3.5	16
22G							0	4	5.5	8.5	9	10	12.5	15	13.5	11	6.5	7	14.5
23H								0	4	7	5	8	11.5	15	13	10.5	6	11	15
24I									0	3	5.5	5	7.5	10.5	9	6.5	2	11	11.5
25J										0	6	2	4.5	8	6.5	4.5	2.5	13.5	9.5
26K											0	5	9.5	13.5	12	10	6.5	15.5	15
27L												0	4.5	8.5	7.5	5.5	5	15.5	10.5
28M													0	4	3.5	3	6	16.5	6.5
29N														0	2	4	9	17.5	3.5
30O															0	2.5	7	15.5	3
31P																0	5	14	5
32R																	0	11	9.5
33S																		0	16
34T																			0

Since the data used were approximations, the results of the program should be considered only as an experimentation with clustering school districts in order to achieve a specific racial balance and to minimize transportation. When distant places were included in one cluster, that was an indication of an imbalance between the existing capacity and the calculated assignment pattern. The output indicated that in an optimal program:

The capacity of	District 1	would be extended to the use of students from	Districts 8, 17 and 20
The capacity of	District 2	would be extended to the use of students from	Districts 15 and 17
"	3	"	Districts 4 and 17
"	4	"	District 17
"	5	"	Districts 1 and 21
"	6	"	Districts 23 and 25
"	7	"	Districts 3, 5, 9, and 17
"	8	"	Districts 4, 16 and 18
"	9	"	Districts 12, 14 and 17
"	10	"	Districts 11, 12, 13, and 16
"	11	"	Districts 8 and 16
"	12	"	Districts 10 and 12
"	13	"	Districts 10, 13, 14, 17, and 1
"	14	"	Districts 2, 3, 5, and 15
"	15	"	Districts 2, 14, and 15

District	Use Extended
16	Districts 3, 6, and 13
17	Districts 3 and 9
18	Districts 5, 6, 20 and 24
19	Districts 18, 20 and 23
20	Districts 3, 5, 6 and 8
21	District 9
22	District 26
23	Districts 3, 4 and 10
24	District 28
25	District 4
26	Districts 1, 3, 7, 11, 31
27	Districts 2, 6, 7, 19, 27, 30, 32, 33
28	Districts 23, 26, 27
29	District 10
30	Districts 5, 13, 23, 28
31	Districts 4, 10, 12, 23
32	Districts 18, 21, 23
33	Districts 17, 23, 29
34	Districts 11, 14, 16, 17, 22, 25, 28

COMMENTS ON EXSISTING NUMERICAL MEASUREMENTS OF DESEGREGATION

Aided by the Social Research Group at the George Washington University, the Officer of Civil Rights (OCR) within DHEW has developed several numerical systems to measure levels of integration in public schools. Their indices have been sufficiently described and very skillfully analyzed by OCR, the GW Research Group and the major consultant on the subject: The Lambda Corporation. In order to make comments on these indices and introduce them into the model, presented by this study, a brief description and analysis of OCR measures seems desirable.[1]

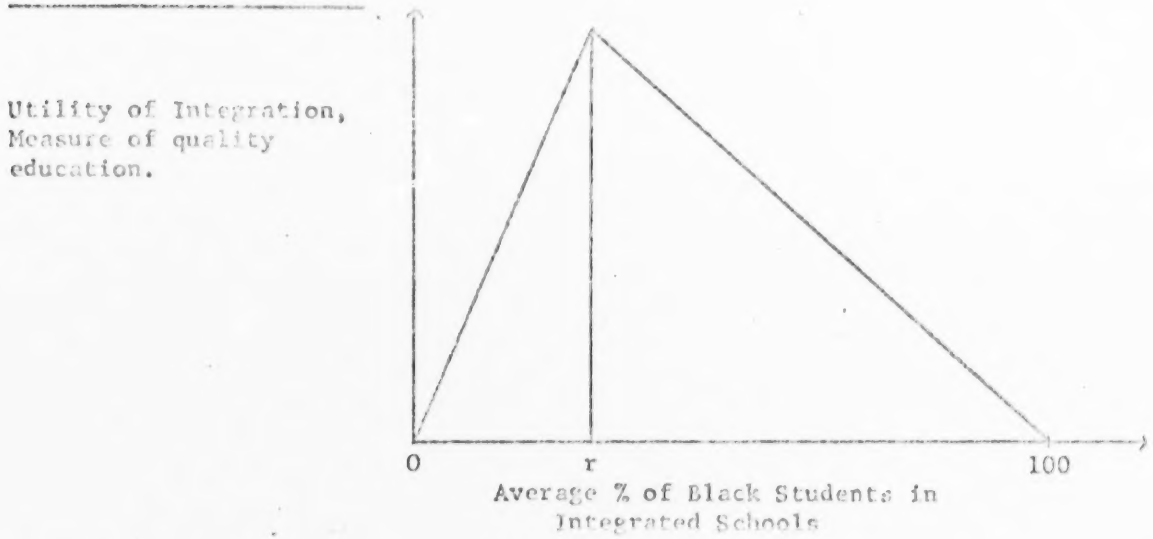
Numbers, which intend to reflect the level of racial integration serve also as measures of achievement towards "quality integrated education" (utility), resulting from student reassignment, attendance area redesign, etc. If utility is a function of the average percent of black children in integrated schools, and we assume that above a certain ratio "r", there is no positive value in integration, the two-space region formed by the domain and range will produce a convex and a concave set.

Note: To fill the gaps between the curt conceptional description and the comments which follow, the reader is referred to the three major documents which may be available at OCR.

[1] Lambada Corporation:
School Desegregation with Minimum Busing, A Report to the Assistant Secretary for Planning and Evaluation; U.S. Department of Health, Education and Welfare, December 10, 1971.

[2] Memorandum of Dr. Constantine Menges to Dr. George Pugh on "Criteria for Measurement of Integration Level", September 29, 1971.

[3] Technical Memorandum by Ira H. Cisin (Social Research Group, The George Washington University) to OCR, entitled "Statistical Indices of School Integration", January, 1970.



Where $0 \rightarrow r$ is the convex segment of domain
$r \rightarrow 100$ is the concave segment of domain

Notation

P_t = percent of Black Students in Study Area S

P_i = percent of Black Students in the i^{th} school district S_i of S.

N_t = total student population in S.

N_i = student population in S_i.

K = number of schools in S.

$S = \{S_i\}$ connected, exhaustive and ∂S_i is empty.

$\sum_{i=1}^{K} S_i = S$, (or K school districts make up S)

$P_i \ldots P_k$ are variables, whereas P_t, N_t, N_i and K are considered constants. (Eventually, a change in P_i will cause a corresponding change at least in one P_j $(i \neq j)$. That, however, can be written as $\triangle P_j = P_j(\triangle P_i)$, but $\triangle P_i = P_i\ (P_i)$.).

There are eight non-concave utility measures:

1. Weighted Average Percentage of White Students by Districts, Using the Number of Black Students by Districts as Weights. (H_1)

$$\frac{\sum_{i=1}^{K} N_i P_i (1 - P_i)}{N_t P_t} = H_1(P_i \ldots P_k)$$

$$\text{The sum of weights } \sum_{i=1}^{K} N_i P_i = N_t P_t$$

In all black districts $P_i = 1$ and in an all white district $P_i = 0$. Both cases will produce zero terms.

Each term of H_1 (school district, school or class-room) contributes to integration in direct proportion to the number of minority students and the percentage of whites. Or each black child contributes $(1 - P_i)$ to the integration measure. If Mosby Circle Elementary School has 40% black children among its 100 total student population, then each of the 40 black children will contribute $(1 - .40 = .60$ to the integration measure. The term in H_1 is calculated as $100(.4)(.6) = 24$.

Consider another single term in H_1: $N_i P_i (1 - P_i) = h_i$ The maximum value of the expression, subject to changes in P_i can be written as,

$$\frac{dh_i}{dp_i} = N_j - 2N_i P_i$$

This derivative should be set equal to zero to find its maximum value. (The reader may notice that the second derivative of h_i with regard to P_i is negative which provides the sufficient condition for the maximum value of the function.) Since $N_i(1 - 2P_i) = 0$, $P_i = .5$ That ratio could not be attained for all schools where P_t .5. Maximum value for Study Area therefore could be attained if all K schools had a uniform

50% minority population, a condition much in conflict with the definition of "minority". This warrents a somewhat deeper investigation of H_1's maximum point. Using the Lagrange multiplier method we can say, that:

$$\mathcal{U}(P_1 \cdots P_k, \lambda) = H_1 + \lambda\left(\sum_{i=1}^{k} P_i N_i - P_t N_t\right)$$

Computing the partial derivatives;

$$\frac{\partial \mathcal{U}}{\partial P_i} = (N_i - 2N_i P_i) + \lambda N_i \qquad \text{and}$$

$$\frac{\partial \mathcal{U}}{\partial \lambda} = \sum_{i=1}^{K} P_i N_i - P_t N_t$$

Equating them to zero,

$P_i = \frac{1+\lambda}{2}$ and $\sum_{i=1}^{K} P_i N_i = P_t N_t$ will determine the condition of maximum for H_1. Since we want to maximize for the entire Study Area, all P_i's will have to be $\frac{1+\lambda}{2}$. Therefore $P_i = P_j$ and $P_t N_t = \sum_{i=1}^{K} P_j N_i = P_j \sum_{i=1}^{K} N_i$ so that $P_t N_t = P_j N_t$ or $P_t = P_j$ for all schools.

Thus our function is maximized when H assumes the value of

$$\frac{\sum N_i P_i (1 - P_t)}{N_t P_t} = 1 - P_t$$

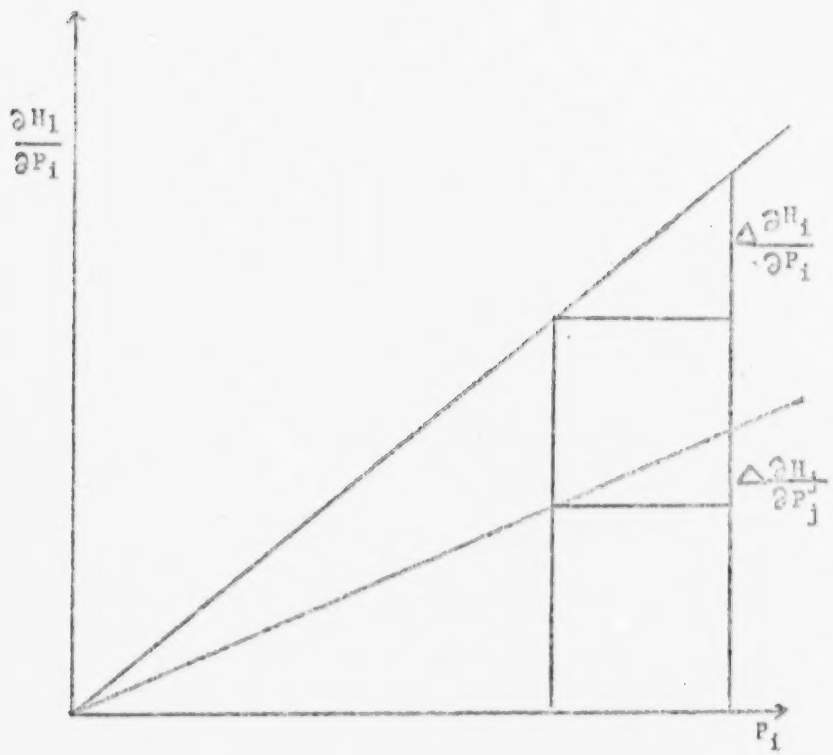
Accordingly, the student is the best off in a class of exactly even racial composition. Utility, on the other hand, for the entire study area is the greatest when the ratio of blacks in each school district is the same as their ratio in the community. This controversy may be resolved by considering the students of the minority race as scarce resource which, however, does not solve the problem of desegregation and busing. The reader should keep in mind that integration measures represent only one aspect of a much more complex social welfare function. Therefore, integration measures should be used

only in relative and never in absolute context. Maybe even further qualifications will have to be made.

The large number of significant factors unheeded by the utility function containing solely integration measures and excluding other factors such as the quality of education, harmful racial tension resulting in physical violence and neurotic disturbances, to mention just a few. In view of all these, it is questionable whether interregional comparisons of desegregation efforts based exclusively on integration measures can be meaningful.

H_1 changes linearly with any alteration in P_i.

$$\frac{\partial H_1}{\partial P_i} = -\frac{2P_i N_i}{N_t P_t} \qquad \text{where } i = 1 \ldots . n$$



The slope depicting the rate of changes depends, if increase in P_i is assumed to be uniform for two distinct school districts, upon the size of the respective districts as measured by N_i and M_j;

$$\triangle \frac{\partial H_1}{\partial P_i} \Big/ \triangle \frac{H_i}{P_j} = N_i / N_j$$

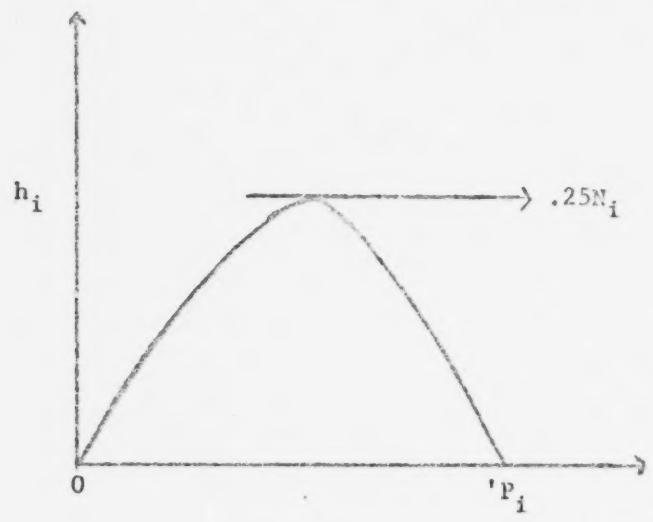
If all P_i's are the same they must equal to P_t. In that case the first derivative, as well as the entire problem, becomes meaningless. The investigation of changes in more than one P_i's calls for the total derivative:

$$dH_1 = \frac{1}{N_t P_t} \left[N_1 dP_1 - 2P_1 N_1 dP_1 + N_2 dP_2 - 2P_2 N_2 dP_2 + \ldots + N_n dP_n - 2P_n N_n dP_n \right]$$

$$dH_1 = \frac{1}{N_t P_t} \left[(N_1 - 2P_1 N_1) dP_1 + (N_2 - 2P_2 N_2) dP_2 + \ldots + (N_n - 2P_n N_n) dP_n \right]$$

A uniform change in P_i, let us say a change of one student reassigned in each district, simplifies the equation to $\frac{1 - 2P_t}{P_t}$. This confines increases in H_1 to $P_t < .25$, per student.

The total increase for a district with N_i students will result in a maximum h_i of $.25N_i$. This can be seen from the fact that $h_i = N_i \; .50(1 - .50) = .25N_i$.



Desegregation Index H_1 computed by districts.

District	P_iN_i	$1 - P_i$	$P_iN_i(1 - P_i)$
1	1151	.637	733
2	1153	.667	769
3	1159	.609	706
4	1138	.616	701
5	1040	.681	708
6	313	.793	248

$P_iN_i = 5954$ $\qquad$ $3865 = P_iN_i(1 - P_i)$

$$H_1 = \frac{P_iN_i(1 - P_i)}{P_iN_i} = \frac{3865}{5954} = .65$$

Notice the inherent weakness of desegregation indices computed by districts; a complete segregation within each district is possible while maintaining the indicated level of utility.

2. The Normalized Version of H_1 denoted ϕ_1.

$$\frac{\sum_{i=1}^{K} N_i P_i (1 - P_i)}{N_T P_T (1 - P_T)} = \phi_1 (P_1 \ldots P_k)$$

This measure has a 0 to 1 range which makes comparisons between school systems feasible and evaluation more convenient.

The maximum value of "1" is reached when $P_i = P_T$ for $i = 1 \ldots k$, or when in each school (or in each class-room), the ratio of black students is the same as the average ratio of black students in the study area.

When all $P_i = P_T$,

$$\phi_1 (P_1 \ldots P_k) = \frac{\sum N_i P_T (1 - P_T)}{N_T P_T (1 - P_T)} = \frac{\sum N_i P_T - P_T \sum N_i P_T}{N_T P_T (1 - P_T)} =$$

$$= \frac{N_T P_T - P_T N_T P_T}{N_T P_T (1 - P_T)} = 1$$

ϕ_1 indicates that up to what percent did the school system approach the ideal, maximum of $1 - P_T$, under a given desegregation plan.

3. The Mean Square Racial Deviation. (H_2)

The basic assumption of this model is that the change of this utility (V_i) in segregation is a linear function of the difference (d_i) between the existing black ratio (P_i) and the average black ratio (P_T) for any child in a given school i. $d_i = \left| P_i - P_T \right|$ In other words changes in racial composition will have a direct effect upon the average students utility of "social well-being".

It follows that,

$$\frac{dV_i}{dd_i} = \alpha \cdot d_i$$

Then,

$$V_i = \int_0^{di} (\alpha \cdot d_j) d_j = \frac{\alpha}{2} \cdot d_i^2$$

for all children in a given school,

$$\sum V_i = N_i d_i^2 \quad \text{(if } \alpha \text{ is put equal to 2.)}$$

and for the entire Study Area,

$$V = H_2 = \sum N_i (P_i - P_T)^2$$

If α is the total miasma of social, economic, etc. inter-actions which will equate the deviation $(P_i - P_T)$ with H_2 we do it wisely to exclude it from our formula. This should remind the reader again that desegregation is a very difficult variable to be transformed into quantifiable social welfare. Desegregation indices should be considered as a numerical system to measure racial composition in school systems. The possible functional relationship between desegregation and social welfare should be studied but in the meantime it is advisable to separate the two.

4. The Normalized Version of H_2 denoted ϕ_2.

$$\frac{\sum_{i=1}^{K} N_i (P_i - P_T)^2}{N_T P_T (1 - P_T)} = \phi_2 (P_1 \ldots P_k)$$

This index is very similar to ϕ_1 because simple algebra proves that $\phi_1 + \phi_2 = 1$. Therefore,

$$\phi_1 (P_1 \ldots P_k) = 1 - \phi_2 (P_1 \ldots P_k)$$

and the two measures are in a linear relationship. They will show the

same (although reciprocal) features. ϕ_2 will have a best minimum value of 0, whereas ϕ_1 had a best maximum value of 1.

5. Linear Measure of Racial Deviation (H_3)

$$H_3(P_1 \ldots P_k) = \sum_{i=1}^{k} \left| P_i - P_T \right|$$

H_3 is considered a bad measure by planners and righteously. It can be maximized in a totally segregated system, that is when $P_i = 0$ or 1. If the study area is divided into two parts $N_1 \ldots N_e$ and $N_e + 1 \ldots, N_k$ with $P_i = 0$ for $i \leq e$ and $P_i = 1$ for $i > e$, then

$$\sum_{i=1}^{k} N_i \left| P_i - P_T \right| = \sum_{i=1}^{e} N_i P_T + \sum_{i=e+1}^{k} N_i (1-P_T)$$

and

$$\sum_{i=1}^{e} N_i = N_T(1 - P_T) \quad \text{is the number of majority students.}$$

Similarly,

$$\sum_{i=e+1}^{k} N_i = N_T P_T$$

Hence,

$$\begin{aligned} \sum_{i=1}^{k} N_i (P_i - P_T) &= \sum_{i=1}^{e} N_i P_T + \sum_{i=e+1}^{k} N_i (1 - P_T) \\ &= \left[N_T(1 - P_T)\right] P_T + (N_T P_T)\ (1 - P_T) \\ &= 2N_T P_T (1 - P_T) \end{aligned}$$

The linear measure is extremely susceptible to manipulation, because the bad effect of a segregated school can be counterbalanced in other schools leaving H_3 unchanged. (This is less likely in quadratic forms where significant deviations are so exaggerated that it would be

very difficult to affect them.)

Assuming that it is desirable to have all schools with the same proportion of minority students, the required number of minority students in the i^{th} school is $N_i P_T$. The actual number of minority students there is $N_i P_i$.

$N_i P_i - N_i P_T$ is the number of excess minority students in the i^{th} school or $N_i \left| P_i - P_T \right|$. Adding up for all schools we arrive at H_4. This measure, therefore, has the natural interpretation of the number of students be moved in order to achieve a given level of integration.

This measure implicitly assumes that the "importance" of a deviation from P_T is constant for a given student. The partial derivative of H_3 with regard to P_i is 1 or -1. Disutility decreases by 1 in case of reassigning 1 student to make P_i approach P_T. Disutility increases by one in case of reassigning one student to make P_i get further away from P_T.

6. The Normalized Version of H_3, denoted ϕ_3.

$$\phi_3(P_1 \ldots P_K) = \frac{\sum_{i=1}^{k} N_i \left| P_i - P_T \right|}{2N_T P_T (1 - P_T)}$$

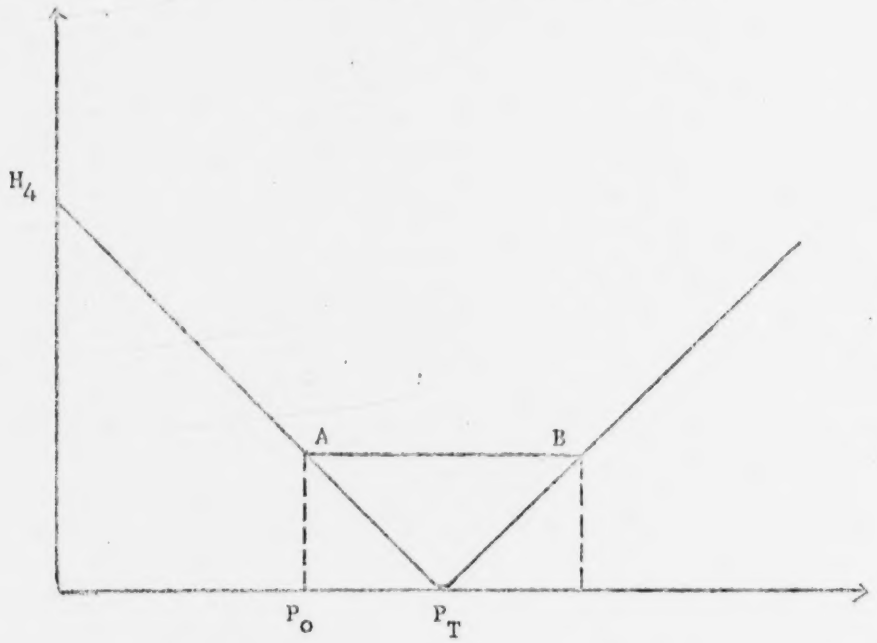
ϕ_3 behaves very similarly to H_3 because both are linear functions. ϕ_3's best and worst values are 0 and 1 respectively.

7. Linear Deviation From Target Composition, H_4.

$$H_4(P_1 \ldots P_k) = \sum_{i=1}^{k} N_i \left| P_i - P_o \right|$$

where P_o is the target composition. When $P_o = P_T$, H_4 is exactly H_3.

Since the minimum value of H_4 is $N_T \left| P_T - P_o \right| > 0$ for $P_T \neq P_o$, there is no unique minimum value for this measure.



In case $P_o < P_T$, both points A and B will satisfy the minimum requirement for one school district. If N = 100, P_o = .20, P_i can be both .30 and .10 to reach the desired value:

$$100 \left| .30 - .20 \right| = 10 = 100 \left| .10 - .20 \right|$$

There are only two possibilities for the two-dimensional (one district) case. The number of minima explodes if two or more schools are taken into consideration. Let us consider another feature of this measure which might open it to manipulative practices. The minimum value of this measure will be reached when all P_i's are the same, P_T so that

$$\min H_4 = N_T \left| P_T - P_o \right|$$

Now let $P_o < P_T$, $P_1 = 1$ (completely segregated black school)

$P_i = P$ for $i = 2, \ldots, K$

We can write,

$$\sum_{i=1}^{k} N_i P_i = N_1(1) + \sum_{i=2}^{k} N_i P = N_T P_T$$

and note that

$$\sum_{i=2}^{k} N_i = N_T - N_1.$$

Therefore, $$P_1 = \frac{N_T P_T - N_1}{N_T - N_1}$$

A non-complying school board therefore may leave a school (or more than one) completely segregated as long as it can find an average P composition for the rest of the schools with $N_T P_T - N_1 > 0$.

It can also be proved that a malpractice like that may have an H_4 equal to the minimum value of H_4. If $P_o < P < P_T$, then H_4 $(\emptyset, P, P, \ldots, P)$.

$$H_4 = N_1(1 - P_o) + (N_T - N_1)(P - P_1) =$$

$$= N_1 - N_1 P_o + (N_T - N_1)P - N_T P_o + N_1 P_o$$

$$= N_1 + N_T P_T - N_1 - N_T P_o$$

$$= N_T P_T - N_T P_o = N_T(P_T - P_o) = \text{Min}$$

This proves that our assumption concerning min H_4 and the segregated subsystem can be numerically upheld.

8. The Normalized Version of H_4, denoted $\emptyset_4$.

$$\emptyset_4(P_1 \ldots P_k) = \frac{H_4}{H_{4\ \max}}$$

shows very similar characteristics to $\emptyset_3$ but has the same flaw as H_4.

II. Non-Convex Measurements.

9. The Tipping Function, H_5.

$$H_5(P_1 \ldots P_k) = \sum_{i=1}^{k} N_i \cdot T(P_i)$$

Where $T(P_i) = P_i$ if $P_i \leq P_o$ and it is zero when $P_i > P_o$. This measurement is based on the assumption that an ideal racial mix is obtained

when there is some specified proportion P_o of black students in a school and any greater ratio is counterproductive. The extent of this counter productivity is estimated, along the non-convex segment, by a contribution to H_5. The maximum value of H_5 is computed by setting as many $P_i = P_o$, as possible. Then, $P_o(N_1 \ldots N_e) = P_T N_T$ if $P_1 \ldots P_c = P_o$ and $P_{e+1} \ldots P_K = 0$. As it will be demonstrated by numerical examples below, $P_T N_T$ can be obtained in highly segregated school systems.

10. The Normalized Version of H_5, denoted $\emptyset_5$.

$$\emptyset_5(P_1 \ldots P_k) = \frac{H_5}{\max H_5}$$

$\emptyset_5$ shows very similar characteristics to H_5.

The Figure on the next page shows graphically the interrelationship among the three most important measures; $\emptyset_1$, $\emptyset_2$, and $\emptyset_3$. As it was observed before, $\emptyset_2$ and $\emptyset_3$ have quite different values on the same set of data. Given values of $\emptyset_2$ (and consequently of $\emptyset_1$) the maximum $\emptyset_3$'s are shown on the left hand scale of the graph. The same also provides for an easy computation of max $\emptyset_3$ given $\emptyset_1$ value. Evidently, $\emptyset_2$ gives an exaggerated measure of the degree of integration, particularly for values of $P_T \leq .2$.

Assuming that $q = \emptyset_2(P_1, \ldots, P_k)$, then the maximum value of $\emptyset_3$ which can be obtained will be one for which it is true that $d_i = |P_i - P_T| = d$, a constant.

Let $d_i = |P_i - P_T|$ and $\emptyset_3(P_1, \ldots P_k) = \bar{\emptyset}_3(d_1, \ldots, d_k)$. Then define $\emptyset(d_1, \ldots, d_k, \lambda) = \bar{\emptyset}_3(d_1, \ldots, d_k) + \lambda(\bar{\emptyset}_2(d_1 \ldots d_k) - q)$. Taking the partials,

$$\frac{\partial \emptyset}{\partial d_i} = \frac{N_i}{2N_T P_T(1 - P_T)} + \lambda \frac{2N_i d_i}{N_T P_T(1 - P_T)}$$

$$\frac{\partial \phi}{\partial \lambda} = \bar{\phi}_2(d_1 \ldots d_k) - q$$

Solving the partials for zero we obtain,

(a) $d_i = -2\lambda - 1$

(b) $\bar{\phi}_2 (d_1 \ldots d_k) = q$

Equation (a) states that all d_i's are equal and equation (b) is the condition that $\phi_2 (P_1 \ldots P_k) = q$, which is the present value of the function. In this case,

$$q = \frac{\sum N_i d^2}{N_T P_T (1 - P_T)} = \frac{d^2}{P_T - P_T}$$

so that

$$d = \sqrt{\frac{q}{(P_T(1 - P_T))}}$$

and

$$\phi_3 (P_1 \ldots, P_k) = \frac{\sum N_i d}{2N_T P_T (1 - P_T)}$$

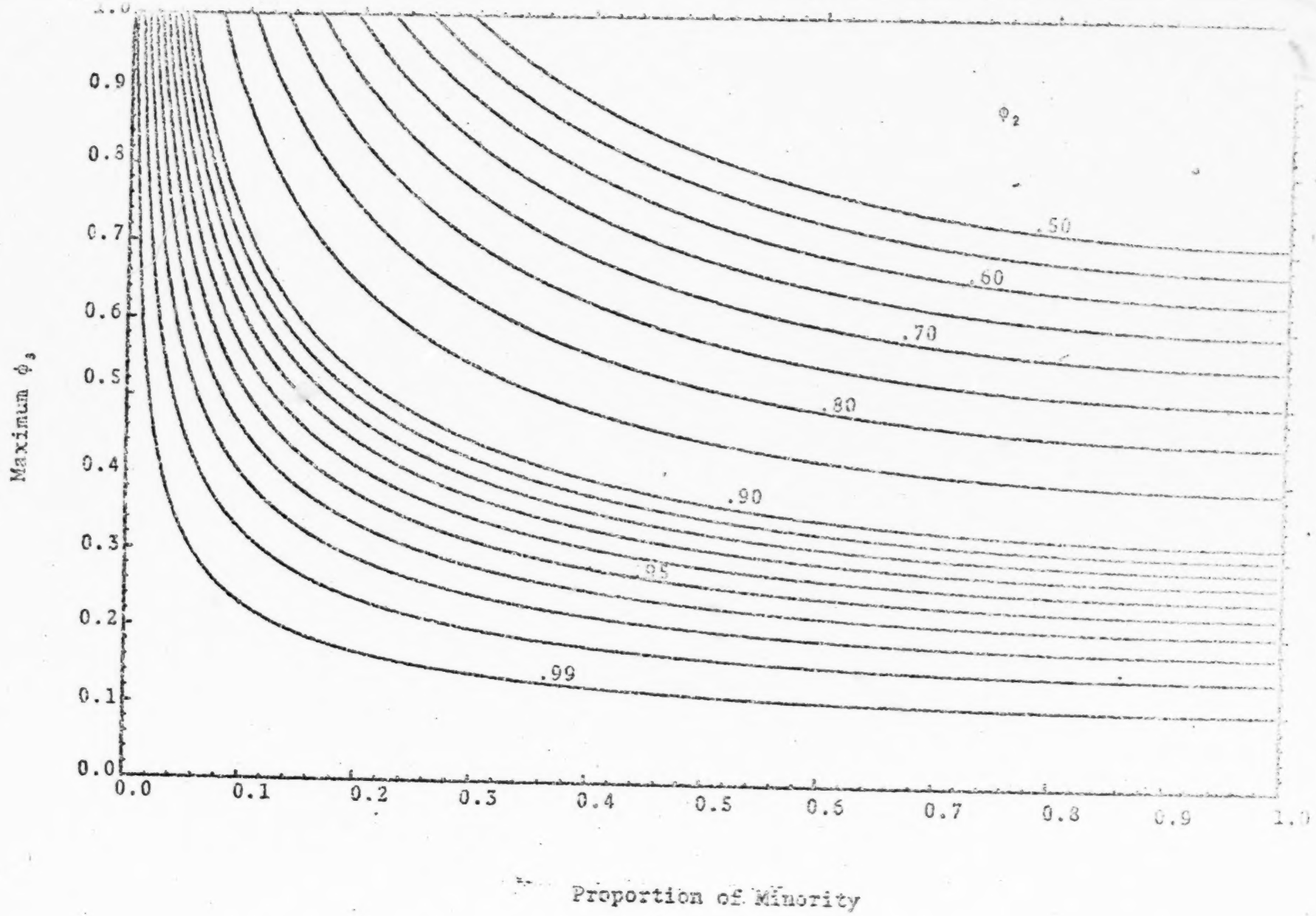
$$= \frac{\sqrt[N_T]{\frac{q}{(P(1 - P_T))}}}{2N_T P_T (1 - P_T)}$$

Therefore,

$$\phi_3 \max = \tfrac{1}{2}\sqrt{\frac{q}{P_T(1 - P_T)}}$$

The minimum value of ϕ_3 is reached when $d_i = 0$. Except for the one at the smallest school, N_1 $(d_1 \neq 0)$ then $\phi_2 (P_1, \ldots\ldots, P_k) = q = \frac{N_1 d^2}{N_T P_T (1 - P_T)}$

or $d = \sqrt{\frac{qN_T P_T (1 - P_T)}{N_1}}$


Maximum ϕ_3
ϕ_2
.50
.60
.70
.80
.90
.95
.99
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
Proportion of Minority

and,

$$\phi_3 \text{ min} = N_1 \sqrt{\frac{\frac{qN_T P_T(1 - P_T)}{N_1}}{2N_T P_T(1 - P_T)}}$$

$$= \tfrac{1}{2} \sqrt{\frac{qN_1}{N_T P_T(1 - P_T)}}$$

$$= \sqrt{\frac{N_1}{N_T}} \cdot \phi_3 \text{ max}$$

To use this graph, first locate the curved line that corresponds to the given value of $\phi_1 = (1 - \phi_2)$. Then draw a vertical line through the point on the absissa which corresponds to P_T in the area under consideration. ϕ_3 max is then read on the vertical scale. For example, to find ϕ_3 max for $\phi_1 = .9$ and $P_T = .2$, we draw a vertical line for $P_T = .2$ and read ϕ_3 max = .39 (approximately). The ϕ_3 min is then computed by multiplying ϕ_3 max by $\sqrt{\frac{N_1}{N_T}}$. If $\frac{N_1}{N_T} = .1$ then $\sqrt{\frac{N_1}{N_T}} = \sqrt{.1} = .32$ and $\phi_3$min =.12. That is, if $\phi_1(P_1, \ldots, P_k) = .9$ and $P_T = .2$, the $\phi_3(P_1, \ldots, P_k)$ is between .39 and .12. Notice that for smaller values of P_T, ϕ_3 max, and consequently ϕ_3 min, increase quite rapidly so that $\phi_2 (P_1, \ldots, P_k)$ is a very poor measure of integration for small values of P_T.

This criterion is reliable and easily implemented, however, the numerical value of F_n has no particular significance. This criteria should not determine the degree or extent of integration. ϕ_3 is more useful for this latter purpose.

A Numerical Example

Assume, that they are three schools, (K = 3) and $N_1 = 50$, $N_2 = 100$, $N_3 = 150$. There are 90 black students in the entire system. $P_T = .30$. P_o (the specified racial composition to be achieved =) .40.

The following table compares $\emptyset_2$, $\emptyset_3$, H_4 and H_5 in their attempt to measure integration. ($\emptyset_1$ had been omitted because $\emptyset_2 = 1 - \emptyset_1$.)

$N_1 = 50$ $N_2 = 100$ $N_3 = 150$

EXAMPLE	P_1	P_1N_1	P_2	P_2N_2	P_3	P_3N_3	$\emptyset_2$	$\emptyset_3$	H_4	H_5
A	.300	15	.300	30	.300	45	.00000	.000	30	90
B	.000	0	.300	30	.400	60	.09524	.238	30	90
C	.000	0	.360	36	.360	54	.07778	.238	30	90
D	.200	10	.400	40	.267	40	.02646	.159	30	90
E	.040	2	.430	43	.300	45	.08048	.206	36	47
F	.440	22	.450	45	.153	23	.10249	.270	44	23
G	.100	5	.400	40	.300	45	.047619	.159	30	90

In example A, $P_i = P_T$ for all schools. This minimizes $\emptyset_2$, $\emptyset_3$ and H_4. Example B is designed to maximize H_5.

In example B the maximum value of H_5 is 90. Looking down the H_5 column notice that this value is achieved for all considered cases with the exception of E and F. Examples B and C have no minority students in school 1. Consequently, the function H_5 does not distinguish very well between highly segregated and desegregated assignments. H_5 is insensitive to segregation because its maximum value is obtained

whenever $P_i \lesssim P_o$ for all i. Thus if $P_T \lesssim P_o$ some schools have no minority students yet the school system can have maximum H_5 measure, as in example B. Consider the following proof to H_5's insensitivity: If $P_1 = 0$ and $P_T(N_T) < P_o(N_T - N_i)$. P_j can be chosen to be $P_j = P_T\left(\frac{N_T}{N_T - N_i}\right)$ $J \neq 1$ and H_5 will reach the maximum value of $P_T N_T$.

Examples B and C demonstrated the weakness of H_4. In fact, the minimum values of $30 = N_T \left| P_T - P_o \right|$ is reached in all but in cases E and F. This is so because the minimum value of H_4 is not zero. There are several patterns of assignments resulting in the same minimum value.

Based upon the above we may say that H_4 and H_5 are unsatisfactory measures of integration. Their insensitivities to large deviation, $\left|P_i - P_T\right| = d_i$, and their susceptibility to manipulation make them unreliable.

The measures $\emptyset_2$ and $\emptyset_3$ are more dependable than H_4 and H_5. (In particular if their values are kept small.) However these functions are also not without flaws.

Example A minimizes $\emptyset_3$ and $\emptyset_2$. In example D there is a reasonable distribution of students (max deviation for $P_T = .3$ is .1) for which $\emptyset_3$ is not zero. However, note that Example G sets forth a highly segregated situation which has the same value for $\emptyset_3$. If $\emptyset_3 = q > 0$ and $q(2N_T P_T(1 - P_T)) > (1 - P_T)(N_T - N_1)$, then the value of q for $\emptyset_3$ can be obtained with $P_1 = 0$ and $P_i = P_T\left(\frac{N_T}{N_T - N_1}\right)$.

Note that $\emptyset_2$ of example D is less than $\emptyset_2$ of example G. $\emptyset_2$ is sensitive to large deviations from P_T. Example B assumes a high degree of segregation, but the $\emptyset_2$ value for B is less than it is for example

F which has a fairly even racial distribution. (The maximum deviation from P_T is .15 in F whereas .30 in B). If the percent of integration is $100(1 - \emptyset_2) = 100(\emptyset\ )$, then in each of the examples except in F, the percent of integration is greater than 90%. In case F the percent of integration is approximately 89%. If $P_T^2 N_1 \leq .1(N_T P_T(1 - P_T)$ one can set $P_1 = 0$ and $P_i = P_T + \sqrt{\frac{.1N_T P_T(1 - P_T) - P_T^2 N_T}{N_T - N_1}}$

to obtain $\emptyset_2 \leq .1$, or what is equivalent, an integration value greater than 90%. This weakness in $\emptyset_2$ can be eliminated by functions which raise deviations $(P_i - P_T) = d_i$ to a power higher than 2.

The One-District Case.

A comparative investigation of the "one district case" reveals several interesting aspects of the applied desegregation indices.

Consider again,

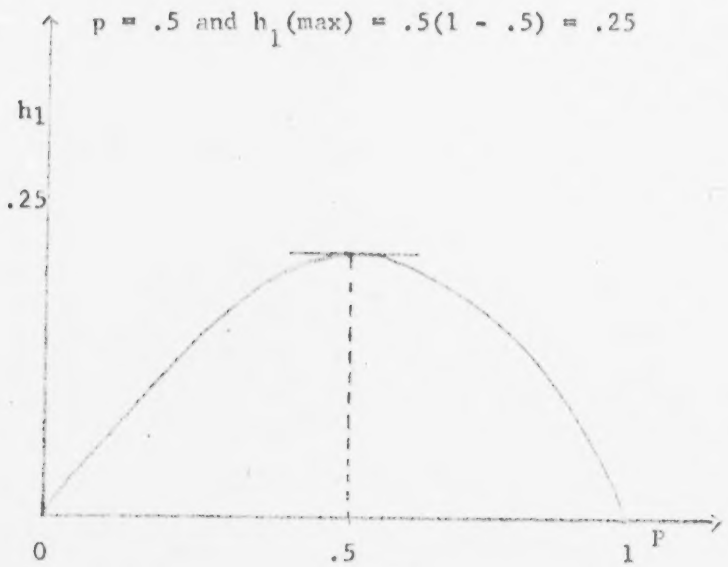
$$h_1 = p(1 - p) = p - p^2$$

$$\frac{dh_1}{dp} = 1 - 2p$$

Finding the maximum point:

$$1 - 2p = 0$$

$p = .5$ and $h_1(\max) = .5(1 - .5) = .25$



Notice that the "sensitivity" of h_1, as a measure, changes with the slope of the curve.

Expressing $p = \frac{1 - x}{2}$ from the derivative of h_1, where $x = \frac{dh_1}{dp}$, we can see that the maximum sensitivity can never reach unit. (When $x \geq 1$, $p \leq 0$. This means that h_1 is a good measure for systems with a relatively small number of black students. In case of a realistic 30% black metropolitan school district, $x = \frac{dh_1}{p} = 1 - 2(.30) = .40$; a one per cent increase or decrease in the district's black children causes a change of .40 in h_1. This shades some serious doubts on the applicability of this measure for inter-regional comparisons.

$$h_2 = (p - p_t)^2$$
$$= p^2 - 2pp_t + p_t^2$$
$$\frac{dh_2}{dp} = 2p - 2p_t$$

Finding the minimum point:

$$2p - 2p_t = 0$$
$$p = p_t$$

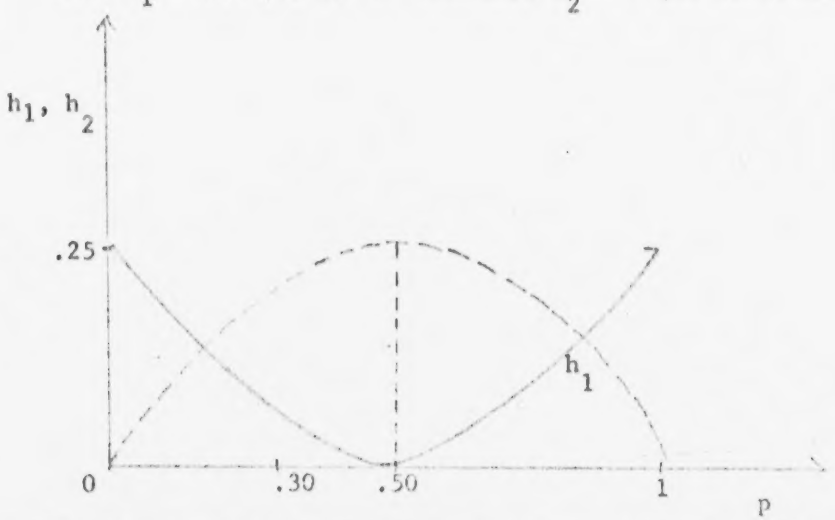
Let us assume that $.50 = p_t$.

Then,

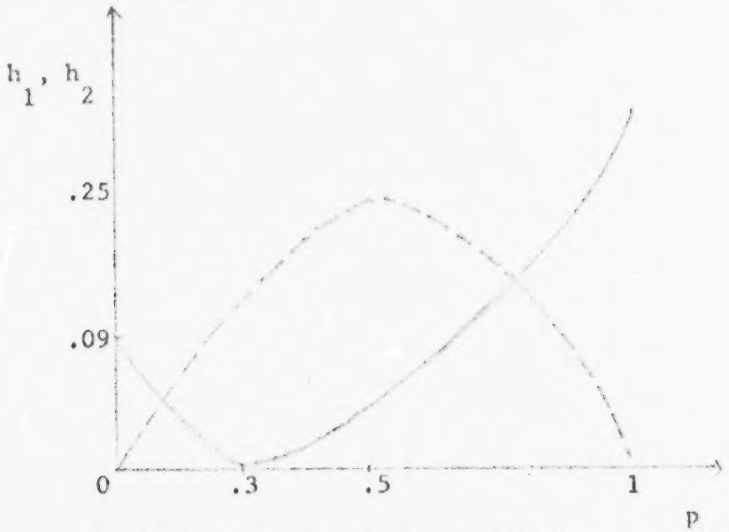
$h_2 = (p - .5)^2 = p^2 - p + .25$

But $p^2 - p = -h_1$, therefore $h_2 = .25 - h_1$

When $h_1 = .25$ (it is at maximum), $h_2 = 0$ (it is at minimum.)



The limited number of black children allows only constrained extreme values for both h_1 and h_2. If $p = .30 = p_t$, $h_1 = .3 - .09 = .21$ is the h_1 (max) and h_2 (min) = 0.



h_2 unlike h_1 might have portions on its graph where its slope exceeds unit, i.e. h_2 becomes very sensitive to changes of racial composition. This sensitivity exists for districts with very high ratio of blacks and where the overall ratio of blacks in the entire school system is small. (This latter makes the second term small.)

For instance at $p_t = .30$

$$\frac{dh_2}{dp} = x = 2p - .6$$

Whenever $p \geq .8$, x will be greater than or equal to one.

Since h_1 and h_2 intersect, there seems to be an indifference in choosing either of the two at and around the point of intercept.

$$h_3 = \left| p - .5 \right| = .5 - p \quad \text{for } p < .5 \text{ and } p - .5 \text{ for } p > .5$$

$$\frac{dh_3}{dp} = -1 \text{ (if } p < .5) \text{ and } +1 \text{ (if } p > .5)$$

When $p = .5$, h_3 has an evident minimum.

One obvious advantage of h_3 in comparison with h_1 is that the slope here is greater than anywhere on h_1.

$h_4 = |p_i - p_o|$ shows features similar to those encountered at h_3. The only difference is that the switch from $\frac{dh_4}{dp} = +1$ and -1 will occure not at p_t but at p_o.

Let us compare the normalized versions. Accepting $p_t = 0$,

$$\emptyset_1 = \frac{p(1 - p)}{.25} = \frac{1}{.25}(p - p^2)$$

$$\emptyset_1' = 4 - 8p$$

The location of maximum point is evidently the same, $p = .50$.

Adding up $\emptyset_1$ and $\emptyset_2$,

$$4p - 4p^2 + 4(p^2 - p + .25) =$$

$$4p - 4p^2 + 4p^2 - 4p + 1 = 1$$

$$\emptyset_1 + \emptyset_2 = 1 \ , \ \emptyset_2 = 1 - \emptyset_1$$

$$\phi_2 = \frac{(p - .5)^2}{.5 \times .5}$$

$$\phi_2' = \frac{(1 - h_1)^2}{.25} = 8p - 4$$

i.e., $\phi_2' = -\phi_1'$

$$\phi_3 = \frac{|p - .5|}{.5} = 2\,|p - .5|$$

$\phi_3' = 2$ if $p > .5$ and $\phi_3' = -2$ if $p < .5$

Since large slope indicates a large sensibility of the measure and ϕ_1' and ϕ_2' are unbounded, they may be considered more desirable than their un-normalized versions, H_1 and H_2.

By setting ϕ_1' or ϕ_2' equal to -2, (ϕ_3', in the realistically minority segment of the curve), we shall draw conclusions on the relative sensibility of these two groups of measures.

$$8p - 4 = -2, \quad p = \tfrac{1}{4}$$

ϕ_1, ϕ_2 and ϕ_3 are equally sensitive at $p = .25$

For smaller p's ϕ_1 and ϕ_2 are more sensitive than ϕ_3. Between a 25% to 50% black student population, the linear measure exhibits a greater sensitivity to variations in racial composition. This, however, does not diminish the shortcomings of the linear measure, pointed out many times by various authors.

i.e. the linear measures divide schools into two groups. One, which has a smaller, (A) and another one (B) which has a greater mean black ratio than the overall average of black students in the system. Or using the expression of the G. W. Research Group,

$$H_3 = \sum_{i \in A} N_i(P_t - P_i) + \sum_{i \in B} N_i(P_i - P_t).$$

All terms within H_3 can be considered constants as long as there is no interaction between group A and B. Further than that H_3 may be subject to infinite manipulation.

New Measures of Integration.

In the course of investigations concerning existing methods of integration and linear programming with racial constraints, we have developed two new models to measure the acceptability of efforts aimed at reducing racial isolation:

I. Specified Deviation from Average Racial Composition, (Γ_n)

II. Interregional Comparison of Desegregation Efforts. (δ)

I. $$F_n = F_n(P_1 \ldots P_k) = \sum_{i=1}^{k} N_i \left| P_i - P_T \right|^n$$

By determining a maximum level of F_n and computing the appropriate "n" value, an even distribution of desegregation efforts within a school system can be assured. It should be emphasized that because of the arbitrary specification of F_n this measure is designed only to evaluate the acceptability of a given student assignment scheme and not to measure integration. Let "d" be the desirable or acceptable and "d_n" the maximum acceptable linear deviation from P_T. For instance if 10% of P_T is the desirable or acceptable and 20% of P_T the maximum deviation from P_T then,

$$d = (.1)P_T$$

$$d_i = \left| P_i - P_T \right|$$

$$d_n = (.20)(P_T)$$

If all schools were within the limits of d, then the largest acceptable value of $F_n(P_1 \ldots P_k) = \sum d^n N_i = N_T \cdot d^n = q$. Letting N_1 be the smallest of the quantities $N_1, \ldots, N_k$, the greatest deviation from P_T is obtained by letting $\left|P_1 - P_T\right| = d_n$ and $\left|P_i - P_T\right| = 0$ for $i = 2 \ldots k$ then $d_n^n \cdot N_1 = F_n$. If we equate this F_n with the above calculated q maximum value, n can be calculated.

$$N_T \cdot d'' = N_1 \cdot d_n''$$

and (bypassing simple algebra),

$$n = \frac{\log N_T - \log N_1}{\log d_n - \log d}$$

$$n = \frac{\log \left(\frac{N_T}{N_1}\right)}{\log \left(\frac{d_n}{d}\right)}$$

The criterion of the acceptability of a given plan of desegregation would then be $F_n \leq q$. If $d = .1 = 1/3(P_T)$, $d_n = .2 = 2/3(P_T)$ (so that $d_n/d = 2$), $N_1 = 50$ and $N_T = 300$ $(N_T/N_1 = 6)$ then by the above formula,

$$n = \frac{\log 6}{\log 2} = 2.585$$

and

$$q = 300(.1)^{2.585} = .7801152$$

The following table is a computation of F_n for the seven examples already discussed;

Example	F_n
A	0
B	2.61515
C	2.39868
D	0.41227
E	2.04945
F	1.36621
G	1.04015

Notice that deviations from the specified d and d_n values are apparent. According to this measure, only situation D is acceptable.

To compare $\emptyset_2$ and F_n, consider $N_T = 50{,}000$, $N_1 = 1{,}000$, $P_T = .3$, $d = .1$ and $d_n = .2$. Then,

$$\emptyset_2 = \frac{d_n^2 N_1}{N_T P_T (1 - P_T)} = \frac{.2 \cdot (1000)}{50{,}000(.3)(.7)} = .0038$$

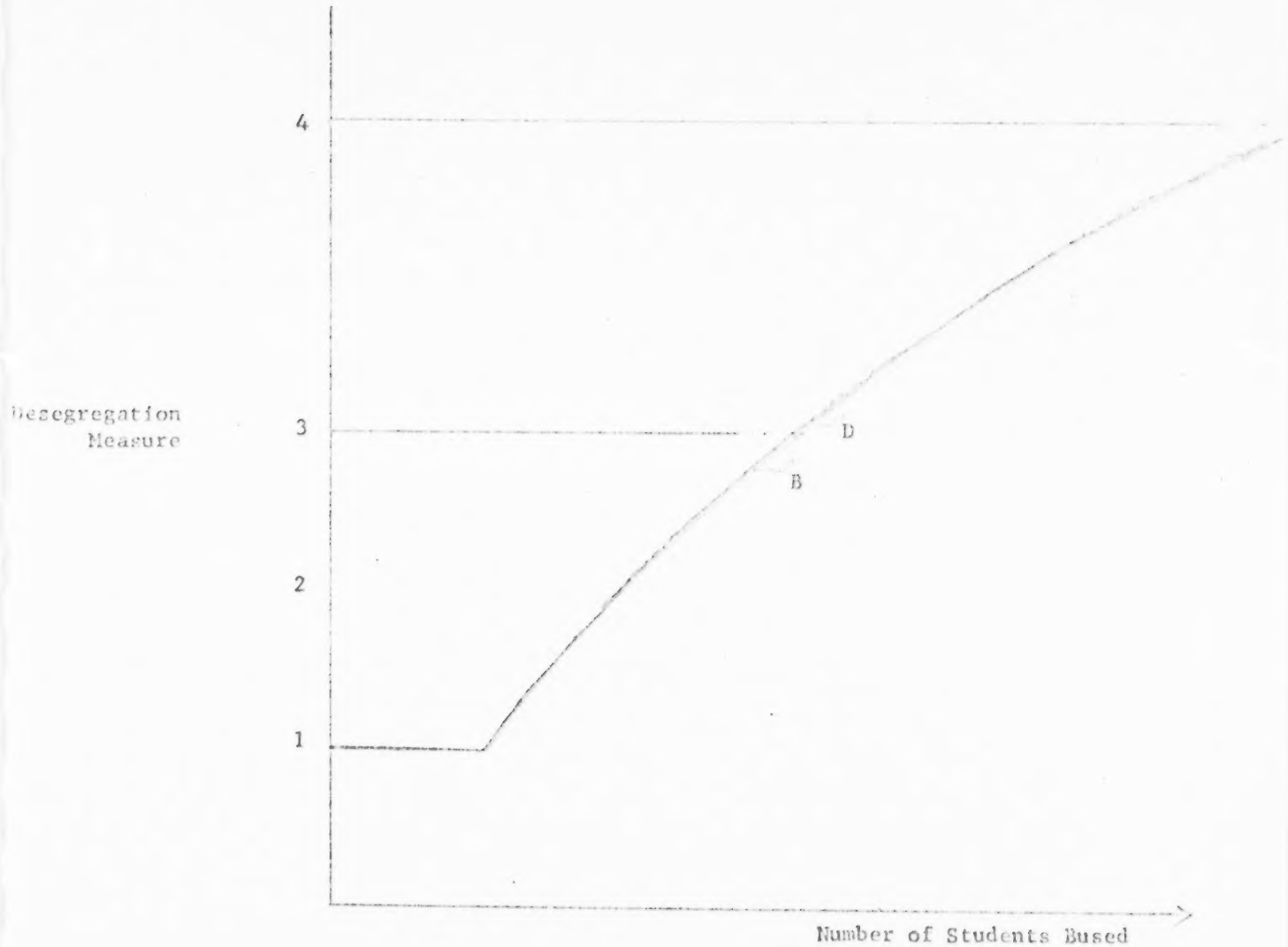
$n = 5.6459$ and $q = .1129$

$\emptyset_2$ may also be limited by a certain value.

11. $$\gamma = \frac{\text{objective function value with racial constraints } (C_c)}{\text{objective function without racial constraints. } (C)} .$$

Given the physically most desirable pattern of commuting in the Study Area (C) a court order or other administrative step will increase the total transportation output. (C_c). The γ ratio considers the effect of the community in the field of desegregation regardless of its site and racial composition. This measure is a side-product of LP.

Desegregation as a Function of Students Bused



Where the slope of this "Desegregation Function" is equal to one, the increase in busing is the same as the increase in desegregation. Evidently, desegregation is more economical below and increasingly uneconomical above that level of busing. Defining the increase in desegregation achieved by "putting an additional student on the bus" as Marginal Desegregation (MD), we can distinguish among four alternative levels of desegregation achieved through busing:

1. Neighborhood School Attendance System to minimize transportation, without regard to the level of desegregation achieved. Assuming a racial polarization between the city and its suburbs, the level of desegregation is dependent on the number (or capacity) of schools and population density in the border zone. This alternative might reduce total transportation in the consolidated area.
2. School Attendance Systems, with a decreasing application of the neighborhood concept, without minimum transportation in the consolidated area but in the "Economical Leg", (MD greater than unit.)
3. School Attendance Systems in the increasingly uneconomical leg, (MD smaller than unit.)
4. Completely Desegregated School Attendance System at "any cost".

Conclusions Concerning Existing Measures of Integration.

A. Indicated favorable levels of integration by existing measures may or may not reflect the true racial composition by schools.

B. The indicated may differ from the actually achieved level of integration for reasons accidental or manipulative.

C. The probable high degree of accidental error gives a dubious value to the use of these integration measures in objective functions which intend to optimize jointly for the level of integration and transportation.

D. In our opinion, $\emptyset_1$, $\emptyset_2$, H_1 and H_2 are the relatively better models because of their sensitivity and freedom from subjective value judgements. $\emptyset_4$, $\emptyset_5$, H_4 and H_5 need modification if ever intended for use and finally $\emptyset_3$ and H_3 are the worst but the easiest perfectable natural measures of integration.

E. All quadratic measures change differently at different racial compositions.

F. None of the described models is apt for the inter-regional comparison of integration levels. This reduces the value of the entire encountered numerical system, since statistically flawless self-measure may not be the primary objective of some school boards.

G. None of the described models considers or compares community efforts to achieve integration.

H. The administrative and data processing costs of the above measures seem to outweigh their benefits.

General Problems of Implementation.

The Expected Volume of Busing

Emotionally motivated public arguments about the "burden of busing", accompanied sometimes by the unwise attitude of a racial cold war, may be restricted to the discussion of facts; if a preliminary survey upon the expected volume of transportation to achieve racial balance was published. In case of consolidating a black majority city core, for instance, with predominantly white suburbs with the intent of assuring some specified, overall racial composition, the number of students to be bused from one municipality to the other may be calculated with relatively small effort. From that number of expected student transportation the annual costs of consolidation may be computed. This calculation will be demonstrated by the so-called Initial Study, The results of such preliminary program would have to be published in order to enable the community to deal with the dilemma of consolidation in rational, numerical terms.

The Expected Scheme of Inter-district Commuting.

Using school districts, which may contain several schools, as destinations and the same districts as origins, a more detailed scheme of cross-city and cross-municipalibly busing could be computed. This will be demonstrated by the so-called Intermidiary Study.

Measuring the level of desegregation achieved by the program.

When a program assigns students to school-districts or schools, the level of desegregation achieved by it should be measured in reliable terms. A very advantageous transportation output maybe a complete waste if the allowed tolerance in desegregation indices is maniputable enough to avoid

desegregation. Therefore, any assignment program aimed at desegregating schools should be accompanied by reliable indices measuring the level of class-room desegregation achieved through increased transportation efforts. It is, however, not recommended to add up desegregation indices and transportation output in the same objective function. For instance, perfect segregation may exist in a completely desegregated school if schools and not classrooms are used as destinations.

The problem of Individual Assignments.

The so-called Final Study will accomplish the detailed assignment of students by grade to the available school capacity. There, destinations will be the available capacities in a given grade by schools and the standard geographic coordinates will represent the origins. This program, however, does not assure a class-room level desegregation and will not decide about the assignment of a given student. The assignment of individual students will have to be performed by a similar program, not demonstrated by this study, where class-room capacities are used as destinations and individual addresses as origins. This computation should be based upon the output of the Final Study which would limit the number of students and schools to be considered by the Individual Assignment Program. This latter is a viable alternative to the use of desegregation indices, since assignments by class-rooms would be performed subject to specified racial compositions.

Specific Problems of Implementation.

Introduction of the Method

Technical assistance for implementation would be the most effective if it came from the Statistics (or related subjects) departments of a local university, or a high caliber consulting firm such as the Lambda Corporation. A less effective method would be the hiring of standard transportation consultants without permanent staff in town. (The technical assistance in question does not include trafficeflow analysis.)

Hiring a statistician for the school board might be the terminal solution to the model adaptation problem.

It is recommended that the provider of technical assistance develop a program for the smallest number of subdivisions like in the "Initial Study" of this report as a demonstration project. A miniature program like that helps the board to develop a practical notion of the model's applicability besides providing basic information about the volume of student transportation. Sensitivity analysis performed on technical coefficients to the extent of variation in the average distance data offers a useful range for the expected total transportation output and provides an opportunity for the board members to voice their opinion which might make them more receptive towards systems analysis.

School Board Resistance

It is clear that a local school board is not geared to change long standing planning methods overnight. The idea of linear programming may succeed if preliminary numerical results of the same are convincing and school board members do not feel that they may be displaced by the introduction of new methods of planning. It is also clear that without knowledge of local schools, the transportation system and prevailing

socio-economic problems in the area, the model can not be meaningfully applied. Outside consultants can provide a meaningful contribution only with the aid of the local school boards. At both the Richmond and Detroit School Boards there was a great deal of interest for systems analytical planning.

Planning Area Layout

The configuration of the planning area and its political subdivisions into districts may have a profound effect on the total transportation requirements of a system.

Changes in Intra-city Transportation Systems

The construction of a new bridge, circumferennial highway, subway or underpass-system might drastically change student assignments. Such a circumstance deserves a careful inclusion in the calculation of technical coefficients despite the well-known difficulties and inaccuracies of dynamic analysis. For instance, if the two years' hence completion of a bridge under construction would alter attendance areas, seemingly it should not be included in the current planning, but we would expect it to be included two years from now. In reality, however, current assignments may remain in force for years if the school board maintains the principle of letting students finish their studies at the school where they started. In such cases, as ever, a viable combination of the systems analytical approach and common sence is recommended.

Computer Programs

Linear programs for the presented model were processed on I.B.M. 370 Mod 145 computer at Virginia Commonwealth University utilizing IBM's "Mathematical Programming System/360, Version 2, Modification 9" as described in IBM Application Program GH20-0476-2. The program is capable of processing jobs with up to 4,095 constraints and objective functions combined; i.e., the number of rows in the matrix must be less than 4,096. These specifications might represent a problem for the school board and its consultant both in terms of computer equipment and qualified personnel. These requirements, however, do not exclude average American cities with university campuses in the area from the possibility of implementation. Besides, there are several other sorts of equipment, including better hardware and programs, which can calculate the linear programming transportation model.

Comments on Legal Matters, Prospects for Implementation.

This study concentrated on the Richmond and Detroit metropolitan areas where busing to achieve racial desegregation is a paramount problem. From experience in dealing with the respective school boards, it seems that the application of systems analytical methods is a viable alternative to present planning practices in school zone delineation. Future implementation depends, to a large measure, upon legal decisions concerning existing plans of desegregation. Considering the present status of those pending legal decisions, prospects for immediate implementation are not promising.

In 1972, U.S. District Judge Robert Merhige ordered the consolidation of the Richmond, Henrico and Chesterfield school districts. The Fourth U.S. Court of Appeals reversed Judge Merhige's order, blocking the implementation of the consolidation plan, which was prepared by the plaintiff, the School Board of Richmond. Defendent, the State Board of Education, acting on behalf of the two municipalities now faced a continuation of the case.

Defendent hired constitutional specialist Phillip Kurland of the University of Chicago to present arguments to the Supreme Court against the consideration of the case. Consideration could have been denied either for reasons of "sui generis" or for the lack of disagreement between the Fourth Court of Appeals and other Court of Appeals rulings on the same subject.

Plaintiff argued that "certiorari" should be granted to the case because the "decision of the Court of Appeals can not be reconciled with either applicable decisions of (the Supreme Court) or the rational of other federal courts in dealing with the related issues".

After petitioning (71 pages) and counter petitioning (21 pages) the Supreme Court decided to hear the case. Judging from the history of the Richmond consolidation battle, lengthy litigations can be expected in every pending and furture consolidation suit.

In Detroit, Judge Steven Roth ordered desegregation between the predominantly black city and its surrounding white suburbs, each of which has prepresented a separate school district thus far. This 1972 decision has been upheld by the 6th Circuit Court of Appeals with one restriction: The consolidation of each suburb will have to be tried separately on District Court level. Since there are 30 suburbs affected by the present plans of consolidation, legal proceedings in Detroit also are likely to be protracted.

Although there are many other interesting cases in the country where the planning problems of consolidation could be studied and new methods tried out, Richmond and Detroit represented the two possible, broad categories of planning problems.

In Richmond, racially balanced contiguous sections were planned to make up the consolidated area, whereas in Detroit the pairing (or clustering) of school districts, in most cases not adjacent, were considered the socially and psychologically proper approach.

This difference in the basic premises determines two distinct directions for further research in the fields of linear programming applications.

Recommendations.

I. Linear Programming should be considered as an alternative systems analytical model to minimize transportation in designing racially balanced subdivisions or clusters of inner districts and suburbs and, in general, to deal with desegregation related planning problems.

II. As a common venture of universities and business firms involved in this field of research, the relative merits of the present model, and other models designed to serve the same purpose, should be tried out on the same data, and under similar conditions, by applying every major available method in one metropolitan area where desegregation related planning problems exist.

III. Existing measures of desegregation should be used very cautiously since those measures are both distortive and may be subject to manipulation.

IV. The γ-ratio and F_n, as new measures of desegregation are recommended for use in mathematical models dealing with minimizing transportation subject to varying racial composition.

V. An application of the subject model for the Richmond metropolitan area to determine a minimum volume of busing, which would be entailed by some plan other than that presently comtemplated by the Richmond School Board, would require about three weeks for the staff of this project.

It is recommended, therefore, that if the issue of consolidation at minimum transportation cost and with the minimum number of students bused emerges in hearings before the U.S. Supreme Court, the subject model be applied to aid in the conceptual evaluation of the Richmond consolidation problem.

Intermediate Grades Student Assignment Problem-Jefferson.

Minimum Transportation Output: 174,664

Description: Increase capacity and number of students to 500

	1		2		3		4		5		6		TOTAL ASSIGNED	ADJUSTMENT
	B	W	B	W	B	W	B	W	B	W	B	W		Z
1B	4 1726		7		8		3		6		16		1726	
1W		3031											3031	
2B	7		4 1723		5		6		7		18		1723	
2W				3467									3467	
3B	9		5		4		1821		9		16		1821	
3W								2620					2620	
4B	9		6		4 1621		9		6		13		1621	
4W						2767		55					2822	
5B	6		7		8		6		4 1560		13		1560	
5W										3332			3332	
6B	16		15		16		13		15		13 466		466	
6W												1801	1801	
TOTAL ASSIGNED	1726	3031	1723	3467	1621	2767	1821	2675	1560	3332	466	1801	25,997	

Cost coefficients printed on form used unless otherwise indicated.